

Online Appendix for "Finance-thy-Neighbor. Trade Credit Origins of Aggregate Fluctuations."

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Appendix A describes the data sources and variables used in Sections 2 and 4. Appendix B derives the equilibrium conditions of the model introduced in Section 3. Appendix C characterizes the equilibrium in the Cobb-Douglas Economy and Appendix D provides proofs of the lemmata and propositions in Section 3.2. The calibration strategy and additional results of the quantitative application are discussed in Appendix E.

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A: Data Sources and Sample Description

GDP and Sectoral Output. Data on nominal and real aggregate US GDP, Gross Output and the GDP-Deflator are obtained from the [Bureau of Economic Analysis \(BEA\)](#). Sectoral data are from the summary tables on "Use of Commodities by Industries After Redefinitions".

Prices and Wages. Data on total hours worked and sectoral prices are obtained from the [Bureau of Labor Statistics \(BLS\)](#), using the MFP- and LPC-Database to deal with missing data when necessary. The *Sectoral Credit Spreads* (r_{it}^z) are derived in [Gilchrist and Zakrajšek \(2012\)](#) and provided to me by the authors.

Balance Sheet and Income Statement Data of US Firms provided by [Compustat](#) database are used to infer sectoral trade credit (shares). A firm is included in the sample if it has

- (1) non-missing NAICS-classification, (2) headquarter in the US, (3) non-missing and non-negative data on balance sheet and income statement items in (a), (4) accounts receivable do not exceed sales, (5) the sum of accounts payable and cash does not exceed total production costs and (6) non-missing consecutive observations in 2005-2010.

In addition, firms are excluded who either enter or exit the Compustat database during the period 2005-2010. A firm's observation is assigned to the previous calendar year if its fiscal year ends in January through May and assigned to the current calendar year otherwise.

The following balance sheet and income statement items are obtained to construct the variables in Section 2 and to calibrate the model in Section 4:

- (a) Accounts Payable (ap), Accounts Receivable (ar), Cost of Goods Sold ($cogs$), Sales (R), Total Assets (at), Current (lc) and Total Liabilities (lt), Debt in Current Liabilities (dlt), Notes Payable (np) and Total Long-Term Debt (dlt).
- (b) Net Income (ni), Dividends (dv), Cash (ch), Interest Expenditures on short- and long-term debt ($xint$), and Interest Income ($idit$), Depreciation associated with spreading the cost of tangible assets over their life (dp)

In case of the variables listed in (b), any missing values are assigned the value of zero and a firm-year observation is excluded if the sum of net income (ni), dividends (dv) and interest expenditures ($xint$) is zero. The sectoral composition of the (I) restricted and (II) unrestricted sample with respect to the time coverage and missing values of US-firms is presented in Table A.1. While 2,697 firms are included in the initial sample per year on average, the reduced sample used to construct Figures 1, 2 contains 816 firms with non-missing values over the entire sample period 1997-2016.

Table A.1

ID	Sector	Description	#Firms		ID	Sector	Description	#Firms	
			(I)	(II)				(I)	(II)
1	11	Agriculture	4	13	24	42	Wholesale Trade	58	145
2	211	Oil and Gas	5	58	25	441	Motor Vehicle and Parts Dealers	7	17
3	212	Mining, except 211	7	26	26	445	Food and Beverage Stores	6	16
4	213	Support for 212	8	21	27	452	General Merchandise Stores	11	22
5	22	Utilities	56	176	28	4A0	Other Retail	8	20
6	23	Construction	12	29	29	481	Air Transport	6	8
7	311T2	Food, Beverages and Tobacco	38	92	30	482	Rail Transport	14	25
8	313T6	Textile, Apparel and Leather	25	69	31	484	Truck Transport	4	26
9	321	Wood Products	6	14	32	486	Pipeline Transport	5	25
10	322T3	Paper Products and Printing	18	46	33	48A9	Other Transport and Warehousing	42	136
11	324	Petroleum and Coal Products	8	22	34	511	Publishing Industries	5	71
12	325	Chemical Products	53	186	35	512	Motion Picture and Sound	1	13
13	326	Plastics and Rubber Products	12	33	36	513	Broadcasting & Telecommunications	13	93
14	327	Nonmetallic Mineral Products	9	21	37	514	Information Services	4	59
15	331	Primary Metals	19	40	38	54	Professional & Technical Services	21	136
16	332	Fabricated Metal Products	28	60	39	55	Management of Companies	-	-
17	333	Machinery	52	131	40	56	Administrative & Waste services	32	86
18	334	Computer and electronic Products	65	274	41	62	Health Care & Social Assistance	23	84
19	335	Electrical Equipment and Components	19	49	42	71	Arts, Entertainment, Recreation	8	32
20	3361MV	Motor Vehicles, Bodies and Parts	25	53	43	72	Accommodation & Food Services	20	79
21	3364OT	Other Transportation Equipment	18	34	44	81	Other Services except GOV	5	15
22	337	Furniture and Related Products	12	24	45	GOV	Government and Education	5	37
23	339	Misc Manufacturing	19	81					

Note: This table reports the NAICS (2007) IDs and descriptions of sectors included in the calibration of the model, and the average number of Compustat-firms (#Firms) in each industry over the entire sample period 1997-2016.

B: Model Discussion and Derivations

B.1. Bank versus Trade Credit

The sector-specific bank interest rate defined in Equation (7) can be micro-founded by introducing a standard perfectly competitive banking sector following the exposition in Freixas and Rochet (2008), Chapter 3. The banking sector features risk-neutral banks with access to unlimited funds, that specialize in lending to a particular sector at interest rate r_i^b for $i = 1, \dots, N$. To finance their lending activity, banks borrow on the interbank market at interest rate r . The banking technology in the form of monitoring costs is subject to sector-specific shocks, z_i , and linear in the amount of loans extended, $C(z, L)$. Omitting the firm's subscripts to simplify notation, the representative bank's problem then equals

$$\max_L \Pi = (1 + r^b)L - (1 + r)L - C(z, L) \quad \text{such that the FOC implies} \quad r^b = r + \frac{\partial C}{\partial L}.$$

The optimal interest rate on bank loans is thus a mark-up over the risk-free rate. Assumption 3 then follows from imposing that the monitoring costs equal a share, A , of the interest costs of financing the amount of loans extended on the interbank market.

$$C(z, L) = \exp(z)A \cdot rL, \quad \text{with} \quad A = (d + \theta^c)^\mu \quad \text{and} \quad \mu > 1.$$

The share, A , is a convex function in the share of sales extended on trade credit by the borrowing firm, θ^c , and the constant d relates to the banking sector's overall monitoring costs independent of a firm's cash-flows. The former accounts for the characteristics of asset-based loans, and trade-finance agreements documented in Lian and Ma (2020) and Ivashina et al. (2022), respectively, and captures that banks extending loans to firms that sell their products using trade credit are indirectly exposed to the credit risk of the firm's customers (Jacobson and von Schedvin, 2015; Costello, 2019).

The previous discussion relates to a strand of literature explicitly modeling financial intermediaries in quantitative macro-models (see i.a. Bernanke et al., 1999). In such models, the equilibrium cost of external finance often includes a risk premium and will be an increasing function of (default) risk in the economy.

B.2. Agent's Optimization Problems and Proof of Proposition 1

For ease of exposition, I first characterize the INTERMEDIATE GOOD PRODUCER i 's total revenues from selling its output and providing trade credit to its customers

$$R_i = \sum_{c=1}^{N+1} (1 - \theta_{ci}) p_i x_{ci} + (1 + r_i^r) \theta_{ci} p_i x_{ci} = \sum_{c=1}^{N+1} (1 + r_i^r \theta_{ci}) p_i x_{ci} = \phi_i^R p_i q_i \quad (\text{B.1})$$

where $c_i = x_{N+1,i}$ and the last equality substitutes the constraints of sector i for production (9) and receivables (10), which are binding in equilibrium. The revenue wedge then equals $\phi_i^R = 1 + r_i^\tau \theta_i^c$, where $\theta_i^c = \sum_c \frac{x_{ci}}{q_i} \theta_{ci}$, with $\theta_{N+1,i} = 0$. The binding working capital constraint implies that total costs of production including interest payments are

$$(1 + r_i^b)BC_i + \sum_s (1 + r_s^\tau) \theta_{is} p_s x_{is} = \phi_i^L W (\ell_i + \ell_i^\tau) + \sum_s \phi_{is}^X p_s x_{is} \quad (\text{B.2})$$

which follows from substituting total bank loans using Equation (3) and defining credit wedges as $\phi_i^L = 1 + r_i^b$ and $\phi_{is}^X = 1 + (1 - \theta_{is})r_i^b + r_s^\tau \theta_{is} \forall s$.

Proof of Proposition 1. (A) The INTERMEDIATE GOOD PRODUCER i 's profit maximization problem (5) is solved as a dual problem in two steps: (1a) For given credit costs and links, let firm i first choose inputs by means of the following cost-minimization problem

$$MC_i Q_i \equiv \min_{\ell_i, \mathbf{x}_i} \phi_i^L W \ell_i + \sum_s \phi_{is}^X p_s x_{is} \quad \text{subject to} \quad Q_i \equiv \mathcal{Q}_i(\ell_i, \mathcal{X}_i(\mathbf{x}_i))$$

where MC_i denotes the marginal cost of one unit of Q_i (including credit costs). The FOCs of the corresponding Lagrangian with Lagrange multiplier, λ , imply

$$\frac{\partial \mathcal{L}}{\partial \ell_i} : MC_i = \phi_i^L W \left(\frac{\partial Q_i}{\partial \ell_i} \right)^{-1} \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial x_{is}} : MC_i = \phi_{is}^X p_s \left(\frac{\partial Q_i}{\partial \mathcal{X}_i} \frac{\partial \mathcal{X}_i}{\partial x_{is}} \right)^{-1} \quad \forall s, \quad (\text{B.3})$$

since $\lambda = MC_i$. The optimal expenditures on labor and intermediate inputs thus are

$$\frac{\partial \ln(Q_i)}{\partial \ln(\ell_i)} MC_i Q_i = \phi_i^L W \ell_i \quad \text{and} \quad \frac{\partial \ln(Q_i)}{\partial \ln(x_{is})} MC_i Q_i = \phi_{is}^X p_s x_{is} \quad \forall s. \quad (\text{B.4})$$

The elasticities of $\mathcal{Q}_i(\cdot)$ wrt all arguments are constant and Euler's Theorem implies that their sum equals one. Substituting the optimal demand for inputs evaluated at $Q_i = 1$ into the production function, $\mathcal{Q}_i(\cdot)$, yields the marginal cost function, $MC_i \equiv mC_i$, in (20).

(1b) Having derived the cost-minimizing input expenditures, firm i then solves for the profit-maximizing level of output choosing, Q_i , for given credit costs and links.

$$\max_{Q_i} \pi_i = \phi_i^R p_i (A_i Q_i)^{\chi_i} - MC_i Q_i - (1 + r_i^b) M_i \quad \text{such that} \quad MC_i = \phi_i^R p_i \left(\chi_i \frac{q_i}{Q_i} \right) \quad (\text{B.5})$$

where $M_i \equiv m_i(\theta_i) = W \ell_i^\tau$. Combining Equations (B.3) and (B.5) yields the firm's optimality conditions (14) and (15) in the main text.

In the final two steps, firm i chooses the optimal trade credit shares (2a) demanded from suppliers, $\{\theta_{is}\}_s$ and (2b) extended to customers, θ_i^c , while taking the demand for trade credit from

their customers, $\{\theta_{ci}\}_c$, as given. The corresponding Kuhn-Tucker Lagrangian is

$$\mathcal{L} = \phi_i^R p_i q_i - MC_i Q_i - (1 + r_i^b) M_i + \sum_s \lambda_s (1 - \theta_{is}) + \nu (1 - \theta_i^c)$$

where $\lambda_s \forall s, \nu$ denote the Lagrange multipliers associated with the feasibility constraints.

(2a) The Kuhn-Tucker Conditions with respect to θ_{is} and λ_s include

$$\frac{\partial \mathcal{L}}{\partial \theta_{is}} = - \left\{ \frac{\partial MC_i}{\partial \theta_{is}} Q_i + (1 + r_i^b) \frac{\partial M_i}{\partial \theta_{is}} \right\} - \lambda_s \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \lambda_s} = (1 - \theta_{is}) \geq 0 \quad \forall s. \quad (\text{B.6})$$

The non-linearity of the credit management cost function and its parameter choices ensure that $\lambda_s = 0$. The optimal credit share then satisfies, $\theta_{is} \in (0, 1)$, and is determined by

$$\frac{\partial M_i}{\partial \theta_{is}} = MC_i \frac{\partial \ln(Q_i)}{\partial \ln(x_{is})} \frac{(r_i^b - r_s^\tau)}{\phi_{is}^X} \frac{Q_i}{1 + r_i^b}, \quad \text{where the RHS follows from Lemma B.1.}$$

Using Equation (B.3) for further simplification, Equation (16) in the main text follows.

To confirm that the optimal credit shares, $\theta_{i\cdot}$, yield a maximum, the SOD of π_i wrt θ_{is} evaluated at the optimum using the FOC and Lemma B.1 implies that

$$\frac{\partial^2 \pi_i}{\partial \theta_{is}^2} = (1 + r_i^b) \left\{ \left[1 - \frac{\partial \ln(Q_i)}{\partial \ln(x_{is})} \right] \frac{(r_i^b - r_s^\tau)}{\phi_{is}^X} \frac{\partial M_i}{\partial \theta_{is}} - \frac{\partial^2 M_i}{\partial \theta_{is}^2} \right\} < 0 \quad \text{since} \quad \frac{\frac{(r_i^b - r_s^\tau)}{\phi_{is}^X} \frac{M_i'}{M_i}}{\left[1 - \frac{\partial \ln(Q_i)}{\partial \ln(x_{is})} \right]^{-1}} < 1.$$

at the optimum and the optimal credit share maximizes profits. Note that while the regularity conditions on $M_i(\theta_{i\cdot})$ are such that $M_i'' > 0$, the marginal cost function is decreasing (increasing) in the credit share if the interest differential is positive (negative) at a diminishing rate, $MC'' < 0$, as shown in Lemma B.1.

(2b) The Kuhn-Tucker Conditions with respect to θ_i^c and ν include

$$\frac{\partial \mathcal{L}}{\partial \theta_i^c} = \frac{\partial \phi_i^R}{\partial \theta_i^c} p_i q_i - \frac{\partial MC_i}{\partial \theta_i^c} Q_i - \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} M_i - \nu \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial \nu} = (1 - \theta_i^c) \geq 0. \quad (\text{B.7})$$

By Assumption B.1, the marginal cost function is convex in θ_i^c such that, $\nu = 0$, and the profit maximizing credit share has an interior solution $\theta_i^c \in (0, 1)$. Equation (17) in the main text follows from substituting the derivative of the marginal cost function in Equation (B.7)

$$r_i^\tau = \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} \left\{ \left[\frac{1}{\phi_i^L} \frac{\partial \ln(Q_i)}{\partial \ln(\ell_i)} + \sum_s \frac{(1 - \theta_{is})}{\phi_{is}^X} \frac{\partial \ln(Q_i)}{\partial \ln(x_{is})} \right] MC_i Q_i + M_i \right\} \frac{1}{p_i q_i} = \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} \frac{BC_i}{p_i q_i} \quad (\text{B.8})$$

where the second equality uses Equations (B.4) and (3). The SOC for a maximum is

$$\frac{\partial^2 \pi_i}{\partial (\theta_i^c)^2} = - \left\{ \frac{\partial^2 MC_i}{\partial (\theta_i^c)^2} Q_i + \frac{\partial^2 \mathcal{R}_i}{\partial (\theta_i^c)^2} M_i \right\} = - \left\{ \frac{\partial^2 MC_i}{\partial r_i^b \partial \theta_i^c} Q_i + \left[\frac{\partial MC_i}{\partial r_i^b} Q_i + M_i \right] \frac{(\mu - 1)}{\theta_i^z} \right\} \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} < 0$$

The last equality follows from the properties of the marginal cost function in Lemma B.1 and Assumption B.1. Then the optimal share, θ_i^c , maximizes profits.

(B) The FOCs corresponding to the HOUSEHOLD's problem, $\max_{C,L} U(C, L)$ s.t. the budget constraint (8), and the final good firm's optimal input demand yield Equation (18) in the main text. The regularity conditions on $U(C, L)$ ensure that the SOC's for a maximum are satisfied.

(C) The FINAL GOOD PRODUCER's profit maximization problem is solved as a dual problem: (1) First, the representative firm chooses the optimal amount of the sectoral inputs $\mathbf{c} = [c_1, \dots, c_N]$ to minimize the cost of producing $F = 1$. Let $\mathcal{P}F$ denote the cost-minimizing expenditures on inputs solving $\mathcal{P}F \equiv \min_{\mathbf{c}} \sum_i p_i c_i$, where \mathcal{P} is the price index of the input composite. The FOCs of the corresponding Lagrangian with Lagrange multiplier, λ , are

$$\frac{\partial \mathcal{L}}{\partial c_i} : p_i = \lambda \frac{\partial \mathcal{F}}{\partial c_i} \quad \text{such that} \quad \mathcal{P} = p_i \left(\frac{\partial \mathcal{F}}{\partial c_i} \right)^{-1} \quad (\text{B.9})$$

since $\lambda = \mathcal{P}$. The properties of the production function ensure that the SOC's for a minimum are satisfied. (2) Next, $\max_F PF - \mathcal{P}F$, yields $P = \mathcal{P}$ and Equation (19) follows. \square

Lemma B.1. *The marginal cost function characterized in Equation (20) in Corollary 1 exhibits positive and decreasing returns in interest rates, $MC' > 0$, $MC'' < 0$, and decreasing returns in credit shares that are negative (positive) if the interest differential between the bank and trade credit rate is positive (negative), $MC' \leq 0$, $MC'' < 0$.*

Proof of Lemma B.1. For given credit costs, $\mathbf{r}_i = [r_i^b, r_1^\tau, \dots, r_N^\tau]'$, and shares, $\boldsymbol{\theta}_i = [\theta_{i1}, \dots, \theta_{iN}]'$, the marginal cost function of sector i is characterized in Corollary 1. Let $x \in \{\ell_i, x_{i1}, \dots, x_{iN}\}$ and $\phi_x \in \{\phi_i^L, \phi_{i1}^X, \dots, \phi_{iN}^X\}$ for ease of notation in the following. All else equal, the derivatives with respect to $v \in \{r_i^b, r_s^\tau, \theta_{is}\}$ are

$$\frac{\partial MC_i}{\partial v} = MC_i \left\{ \sum_x \frac{\partial \ln(Q_i)}{\partial \ln(x)} \frac{1}{\phi_x} \frac{\partial \phi_x}{\partial v} \right\} = \begin{cases} > 0 & v = r_i^b, r_s^\tau \\ \leq 0 & v = \theta_{is} \end{cases} \quad (\text{B.10})$$

The second order derivatives are then calculated as

$$\frac{\partial^2 MC_i}{\partial v^2} = - \frac{MC_i}{v^2} \left\{ \sum_x \frac{\partial \ln(Q_i)}{\partial \ln(x)} \left(\frac{\partial \ln(\phi_x)}{\partial \ln(v)} \right)^2 - \left(\sum_x \frac{\partial \ln(Q_i)}{\partial \ln(x)} \frac{\partial \ln(\phi_x)}{\partial \ln(v)} \right)^2 \right\} < 0 \quad (\text{B.11})$$

which are negative due to Jensen's Inequality.

Lastly, the derivative of marginal costs, MC_i , with respect to θ_i^c follows from

$$\frac{\partial MC_i}{\partial \theta_i^c} = \frac{\partial MC_i}{\partial r_i^b} \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} > 0 \quad \text{and} \quad \frac{\partial^2 MC_i}{\partial (\theta_i^c)^2} = \left[\frac{\partial^2 MC_i}{\partial (r_i^b)^2} \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} + \frac{\partial MC_i}{\partial r_i^b} \frac{(\mu - 1)}{\theta_i^z} \right] \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} > 0 \quad (\text{B.12})$$

where the last inequality follows from Assumption B.1 discussed below and the properties of

the bank rate in Assumption 3. In particular, changes in the share of revenues from product sales extended on trade credit, θ_i^c , affect sector i 's interest rate on bank credit as follows

$$mZ_i \equiv \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} = \mu \frac{r_i^z}{\theta_i^z} > 0, \quad \text{and} \quad mZ'_i \equiv \frac{\partial^2 \mathcal{R}_i}{\partial (\theta_i^c)^2} = \frac{(\mu - 1)}{\theta_i^z} \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} > 0 \quad (\text{B.13})$$

where $\theta_i^z = (d + \theta_i^c)$. Thus, marginal costs are increasing and - by Assumption B.1 - convex in θ_i^c such that a firm's profit maximization problem wrt the extension of trade credit is well defined. \square

Assumption B.1. *The convexity of the interest premium, r_i^z , in θ_i^z in Equation (7) is*

$$\mu > \max_{i \in \mathcal{V}} \left\{ \left[1 - \left(1 - \mathcal{B}_i - \frac{\partial \ln(\mathcal{B}_i)}{\partial \ln(r_i^b)} \right) \frac{r_i^z}{r_i^b} \right]^{-1} \right\} > 1 \quad \text{where} \quad \mathcal{B}_i = \sum_x \frac{\partial \ln(\mathcal{Q}_i)}{\partial \ln(x)} \frac{\partial \ln(\phi_x)}{\partial \ln(r_i^b)} \quad (\text{B.14})$$

with $x \in \{\ell_i, x_{i1}, \dots, x_{iN}\}$ and $\phi_x \in \{\phi_i^L, \phi_{i1}^X, \dots, \phi_{iN}^X\}$.

Derivation of Assumption B.1. Let $x \in \{\ell_i, x_{i1}, \dots, x_{iN}\}$ and $\phi_x \in \{\phi_i^L, \phi_{i1}^X, \dots, \phi_{iN}^X\}$ for ease of notation in the following. Define $\mathcal{B}_i = \sum_x \frac{\partial \ln(\mathcal{Q}_i)}{\partial \ln(x)} \frac{\partial \ln(\phi_x)}{\partial \ln(r_i^b)}$ as the weighted sum of elasticities of credit wedges of sector i wrt bank rates. It follows that \mathcal{B}_i is positive, less than one and increasing in r_i^b at a diminishing rate, where

$$\begin{aligned} \frac{\partial \mathcal{B}_i}{\partial r_i^b} &= \sum_x \frac{\partial \ln(\mathcal{Q}_i)}{\partial \ln(x)} \frac{\partial}{\partial r_i^b} \left(\frac{\partial \phi_x}{\partial r_i^b} \frac{r_i^b}{\phi_x} \right) = \sum_x \frac{\partial \ln(\mathcal{Q}_i)}{\partial \ln(x)} \left[\frac{1}{\phi_x} \frac{\partial \phi_x}{\partial r_i^b} - \left(\frac{1}{\phi_x} \frac{\partial \phi_x}{\partial r_i^b} \right)^2 r_i^b \right] \\ &= \frac{1}{r_i^b} \left[\mathcal{B}_i - \sum_x \frac{\partial \ln(\mathcal{Q}_i)}{\partial \ln(x)} \left(\frac{\partial \ln(\phi_x)}{\partial \ln(r_i^b)} \right)^2 \right] > 0 \end{aligned} \quad (\text{B.15})$$

The second order derivative in Equation (B.12) is positive if

$$0 < \frac{\partial^2 mC_i}{\partial (r_i^b)^2} \frac{\partial \mathcal{R}_i}{\partial \theta_i^c} + \frac{\partial mC_i}{\partial r_i^b} \frac{(\mu - 1)}{\theta_i^z} = \frac{mC_i}{r_i^b \theta_i^z} \left\{ \left[-\mathcal{B}_i + r_i^b \frac{\partial \mathcal{B}_i}{\partial r_i^b} + \mathcal{B}_i^2 \right] \frac{r_i^z}{r_i^b} \mu + \mathcal{B}_i (\mu - 1) \right\}$$

where the second equality follows from combining Equations (B.10), (B.11), (B.13) and (B.15). Condition (B.14) then follows from rearranging.

B.3. Cobb-Douglas Economy

Corollary B.1. *Let the equilibrium conditions be as characterized in Proposition 1 and let the household's utility be given by*

$$U(C, L) = \frac{C^{1-\epsilon}}{1-\epsilon} - \frac{L^{1+\psi}}{1+\psi}, \quad (\text{B.16})$$

where the parameter $\epsilon > 0$ denotes the income elasticity and $\psi > 0$ equals the inverse Frisch elasticity of labor supply. If firms operate with Cobb-Douglas technologies (21) then

(a) the intermediate good producer i 's optimal demand for labor, ℓ_i , and for sector s 's output, x_{is} , satisfy for all $i, s \in \mathcal{V}$

$$\eta_i \chi_i \frac{q_i \phi_i^R}{\ell_i \phi_i^L} = \frac{W}{p_i} \quad \text{and} \quad \omega_{is}(1 - \eta_i) \chi_i \frac{q_i \phi_i^R}{x_{is} \phi_{is}^X} = \frac{p_s}{p_i} \quad (\text{B.17, B.18})$$

(b) the intermediate good producer i 's optimal credit composition to finance intermediate inputs obtained from suppliers, $\theta_{is} \forall s$, and optimal share of revenues extended on trade credit, θ_i^c , for all $i \in \mathcal{V}$ are given by

$$\theta_{is} = \underbrace{\left(1 - \frac{\kappa_{0,is} \bar{\theta}_i^s}{\kappa_{1,is}}\right)}_{\theta_{is}^c} \bar{\theta}_i^s + \frac{(\bar{\theta}_i^s)^2}{\kappa_{1,is}} \frac{(r_i^b - r_s^\tau) p_s x_{is}}{1 + r_i^b} \quad \text{and} \quad r_i^\tau = \frac{\mu r_i^z}{(d + \theta_i^c)} \frac{BC_i}{p_i q_i} \quad (\text{B.19, B.20})$$

The marginal cost function MC_i in Equation (20) becomes

$$MC_i = \left(\phi_i^L\right)^{\eta_i} \left(\prod_s (\phi_{is}^X)^{\omega_{is}}\right)^{(1-\eta_i)} (W)^{\eta_i} \left(\prod_s p_s^{\omega_{is}}\right)^{(1-\eta_i)} \quad (\text{B.21})$$

(c) the household's optimal consumption-labor choice and final good producer's optimal demand for sector i 's output imply for all $i \in \mathcal{V}$

$$\frac{L^\psi}{C^{-\epsilon}} = \frac{W}{p_i} \frac{\beta_i F}{c_i} \quad \text{and} \quad \frac{\beta_i c_j}{\beta_j c_i} = \frac{p_i}{p_j} \quad (\text{B.22, B.23})$$

Proof. Corollary B.1 follows directly from Proposition 1 and Corollary 1. \square

C: Equilibrium Characterization

MATRIX NOTATION & OPERATIONS. Matrices are denoted as bold capital and vectors as bold lower-case letters. Since the economy consists of N intermediate sectors, matrices are of size $(N \times N)$ and vectors are of size $(N \times 1)$, unless otherwise specified. Furthermore, let

$\text{inv}(\cdot)$...	generates element-wise inverse of a matrix
$\text{diag}(\cdot)$...	generates diagonal matrix using vector \mathbf{x}
\circ	...	denotes the Hadamard product

This appendix derives the partial equilibrium of the model economy in Section 3 when firms operate with Cobb-Douglas production technologies (21) and preferences (B.16) while taking the cost and composition of credit as given. Let the aggregate wage W be the numeraire.

C.1. National Accounting and Domar Weights

Revenues, $R_i = \pi_i + (1 + r_i^b)M_i + C_i$, are used to cover dividend payments to the fixed factor, π_i , given by Equation (5) in the main text, credit-management costs including bank interest payments, $(1 + r_i^b)M_i$, and *production costs* (including credit costs)

$$C_i = \phi_i^L W \ell_i + \sum_s \phi_{is}^X p_s x_{is} = (1 + r_i^b)(BC_i - M_i) + \sum_s (1 + r_s^\tau) \theta_{is} p_s x_{is} = \chi_i R_i$$

which follows from the Cobb-Douglas technology and $\sum_s \omega_{is} = 1$.

Total sectoral transfers in the form of dividends and interest rate payments on bank credit are $\pi_i + r_i^b BC_i = (1 - \tilde{\chi}_i)R_i$, where $\tilde{\chi}_i$ is defined in Equation (25).

The *ratio of final revenues to labor income* and the inverse aggregate labor share are

$$\frac{PF}{WL} = 1 + \frac{\sum_i (1 - \tilde{\chi}_i)R_i}{WL} \quad \text{and} \quad \Lambda^{-1} = \frac{PY}{WL} = 1 + (1 - \tau) \frac{\sum_i (1 - \tilde{\chi}_i)R_i}{WL}$$

where the last line follows from taking into account that GDP equals $PY = PF - PH$.

Next, note that the sum of sectoral revenues, R_i , and i 's customers' interest payments on bank-loans obtained to finance input expenditures on i 's output, $T_i = \sum_c r_c^b (1 - \theta_{ci}) p_i x_{ci}$, is

$$R_i + T_i = \sum_c (1 + (1 - \theta_{ci})r_c^b + \theta_{ci}r_i^\tau) p_i x_{ci} + p_i c_i = \sum_c \gamma_{ci} R_c + \beta_i R_{N+1}$$

and $\mathbf{R} = \mathbf{L}'[\beta \mathbf{R}_{N+1} - \mathbf{T}]$ in matrix form, where $R_{N+1} = PF$ denote revenues of the final good producer. Rearranging and dividing by final revenues yields the distortion-adjusted gross Domar weight defined in Equation (26) in the main text.

C.2. Proof of Proposition 2

The following two auxiliary Lemmata C.1 and C.2 characterize equilibrium sales and prices for given interest rates, credit shares and aggregate labor.

Lemma C.1. *The vector of revenues of intermediate good firms is*

$$\mathbf{R} = \tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}} L = \tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}} R_{N+1} \quad \text{where} \quad R_{N+1} = \left(1 + (\iota - \tilde{\chi})' \tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}}\right) L \quad (\text{C.1})$$

denotes the revenues of the final good producer. The Leontief inverses $\tilde{\mathbf{L}}'$ and $\tilde{\mathbf{L}}$ are

$$\tilde{\mathbf{L}}' = [\mathbf{I} - \tilde{\boldsymbol{\Gamma}}' - \tilde{\boldsymbol{\beta}}(\iota - \tilde{\chi})']^{-1} = [\mathbf{I} + \Lambda^{-1} \boldsymbol{\lambda}(\iota - \tilde{\chi})'] \tilde{\mathbf{L}}' \quad \text{and} \quad \tilde{\mathbf{L}} = [\mathbf{I} - \tilde{\boldsymbol{\Gamma}}]^{-1}, \quad (\text{C.2})$$

where $\boldsymbol{\lambda}$ denotes the vector of sectoral Domar weights, and Λ equals the aggregate labor share. The credit-wedge adjusted IO-matrix, $\tilde{\boldsymbol{\Gamma}}'$, the vector of final-demand shares, $\tilde{\boldsymbol{\beta}}$, and of cost shares, $\tilde{\chi}$, are characterized in Definition 2.

Proof of Lemma C.1. (a) The revenues of the *Intermediate Goods Firm* i are

$$R_i = \phi_i^R p_i \left(\sum_c x_{ci} + c_i \right) = \sum_c \frac{\phi_i^R}{\phi_{ci}^X} \gamma_{ci} R_c + \phi_i^R \beta_i \left(WL + \sum_i (1 - \tilde{\chi}_i) R_i \right).$$

The last equality uses the optimal demand for sector i 's output, and takes into account that $R_{N+1} = PF = P(C + H)$. Stacking equations, $\mathbf{R} = [R_1 \cdots R_N]'$, re-arranging, and taking W as the numeraire yields

$$\mathbf{R} = [\mathbf{I} - \tilde{\boldsymbol{\Gamma}}']^{-1} \tilde{\boldsymbol{\beta}} \left(L + (\iota - \tilde{\chi})' \mathbf{R} \right), \quad \text{where} \quad \tilde{\boldsymbol{\Gamma}}' = \text{diag}(\phi_R) (\text{inv}(\Phi_X) \circ \Gamma)'$$

and $\tilde{\boldsymbol{\beta}} = \text{diag}(\phi_R) \boldsymbol{\beta}$ as in Definition 2. The entries of the vector of revenue, ϕ_R , and the matrix of sectoral credit wedges, Φ_X , are characterized in Proposition 1. Further taking into account that R_{N+1} is a function of \mathbf{R} due to DRS, Equation (C.1) follows from noting that $\tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}} = \mathbf{R} (R_{N+1})^{-1}$ and applying the Bartlett inverse to calculate the distortion and income-adjusted Leontief inverse, $\tilde{\mathbf{L}}'$. Lastly, note that

$$\tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}} = \left[\mathbf{I} + \frac{\mathbf{R}(\iota - \tilde{\chi})'}{R_{N+1} - (\iota - \tilde{\chi})' \mathbf{R}} \right] \frac{\mathbf{R}}{R_{N+1}} = \left[1 + \frac{(\iota - \tilde{\chi})' \mathbf{R}}{R_{N+1} - (\iota - \tilde{\chi})' \mathbf{R}} \right] \frac{\mathbf{R}}{R_{N+1}} = \frac{\boldsymbol{\lambda}}{\Lambda}.$$

(b) The revenues of the *Final Good Firm*, $R_{N+1} = WL + \sum_i (1 - \tilde{\chi}_i) R_i$, equal total final consumption sales. Substituting the vector of equilibrium sales yields Equation (C.1). \square

Lemma C.2. (a) *The vector of sectoral prices is*

$$\ln(\mathbf{p}) = \mathbf{L} \left[-\text{diag}(\boldsymbol{\chi})\boldsymbol{\varepsilon} + \ln(\boldsymbol{\phi}) + \text{diag}(\boldsymbol{\iota} - \boldsymbol{\chi}) \ln(\boldsymbol{\lambda}) + (\boldsymbol{\iota} - \boldsymbol{\chi})(\ln(L) - \ln(\Lambda)) \right] \quad (\text{C.3})$$

where $\mathbf{L} = [\mathbf{I} - \boldsymbol{\Gamma}]^{-1}$ equals the Leontief inverse and the vector of wedges is

$$\ln(\boldsymbol{\phi}) = \text{diag}(\boldsymbol{\eta} \circ \boldsymbol{\chi}) \ln(\boldsymbol{\phi}_L) + (\boldsymbol{\Gamma} \circ \ln(\boldsymbol{\Phi}_X))\boldsymbol{\iota} - \ln(\boldsymbol{\phi}_R) \quad (\text{C.4})$$

(b) *The aggregate price level is*

$$\ln(P) = \boldsymbol{\beta}' \ln(\mathbf{p}) = -\ln(Z) + \boldsymbol{\beta}' \mathbf{L} \ln(\boldsymbol{\phi}) + \boldsymbol{\alpha}' \ln(\boldsymbol{\lambda}) + a (\ln(L) - \ln(\Lambda)) \quad (\text{C.5})$$

where $a = \boldsymbol{\alpha}' \boldsymbol{\iota}$ with $\boldsymbol{\alpha}' = \boldsymbol{\beta}' \mathbf{L} \text{diag}(\boldsymbol{\iota} - \boldsymbol{\chi})$ and aggregate productivity, $\ln(Z) = \boldsymbol{\beta}' \mathbf{L} \text{diag}(\boldsymbol{\chi})\boldsymbol{\varepsilon}$.

Proof of Lemma C.2. (a) Substituting for output of sector i in $R_i = \phi_i^R p_i q_i$ using sector i 's production function as given by Equation (21) as well as the profit maximizing intermediate input choices (B.17,B.18) yields

$$R_i = \phi_i^R p_i \left[\exp(\varepsilon_i) \zeta_i \left(\chi_i \eta_i \frac{R_i}{\phi_i^L W} \right)^{\eta_i} \prod_s \left(\gamma_{is} \frac{R_i}{\phi_{is}^X p_s} \right)^{\omega_{is}(1-\eta_i)} \right]^{\chi_i}.$$

Let the aggregate wage rate be the numeraire. Then taking logs and rewriting yields

$$\ln(p_i) - \sum_s \gamma_{is} \ln(p_s) = (1 - \chi_i) \ln(R_i) - \chi_i \varepsilon_i + \eta_i \chi_i \ln(\phi_i^L) + \sum_s \gamma_{is} \ln(\phi_{is}^X) - \ln(\phi_i^R)$$

since $\zeta_i^{-1} = \chi_i [\eta_i]^{\eta_i} [(1 - \eta_i) \prod_s \omega_{is}^{\omega_{is}}]^{1-\eta_i}$. Stacking equations and solving for the optimal price vector using $\mathbf{R} = \tilde{\mathbf{L}}' \tilde{\boldsymbol{\beta}} \mathbf{L} = \Lambda^{-1} \boldsymbol{\lambda} \mathbf{L}$ yields Equation (C.3).

(b) The aggregate price level in Equation (C.5) follows. \square

Proof of Proposition 2. Taking logs of nominal GDP, $PY = \Lambda^{-1} W L$, and substituting the aggregate price index using Equation (C.5) in Lemma C.2 yields Equation (27) in the main text. Equation (29) then follows from the definition of the labor wedge. \square

D: Log-Linearization

In this appendix, I log-linearize the model around its equilibrium, $x = \bar{x} \exp(\hat{x}) \approx \bar{x}(1 + \hat{x})$, where $\hat{x} = d \ln(x) = \ln(x) - \ln(\bar{x})$ denotes the log-deviation of x from steady state, \bar{x} . To simplify notation, I drop the bar-notation in the following while recognizing that the respective elasticities are evaluated at their equilibrium values.

D.1. Auxiliary Lemmata

Auxiliary Lemmata D.1 to D.4 characterize log-changes in credit shares, Domar weights, the aggregate labor income share and sectoral revenues as functions of changes in interest rates and credit shares.

Lemma D.1. *The input-specific credit wedge deviations of sector i are*

$$\hat{\phi}_i^L = \frac{r_i^b}{\phi_i^L} \hat{r}_i^b \quad \text{and} \quad \hat{\phi}_{is}^X = \frac{(1 - \theta_{is})r_i^b}{\phi_{is}^X} \hat{r}_i^b + \frac{\theta_{is}r_s^\tau}{\phi_{is}^X} \hat{r}_s^\tau - \frac{(r_i^b - r_s^\tau)\theta_{is}}{\phi_{is}^X} \hat{\theta}_{is} \quad (\text{D.1})$$

While the sign of the elasticity wrt to credit shares depends on the interest-rate differential $(r_i^b - r_s^\tau)$, the remaining elasticities are positive. The log-linearized revenue wedge is

$$\hat{\phi}_i^R = \frac{r_i^\tau AR_i}{R_i} \hat{r}_i^\tau + \sum_c \frac{r_i^\tau AR_{ci}}{R_i} \hat{\theta}_{ci} + \frac{r_i^\tau}{\phi_i^R} \underbrace{\sum_c \theta_{ci} \mathcal{X}_{ci}}_{=S_{\mathcal{X},i}} \hat{\mathcal{X}}_{ci}, \quad \text{where } \mathcal{X}_{ci} = \frac{x_{ci}}{q_i}. \quad (\text{D.2})$$

Proof of Lemma D.1. Auxiliary Lemma D.1 follows from the log-linearization of the *labor and intermediate credit wedges* of sector i defined in Equations (12) and (13), respectively, in the main-text. \square

Lemma D.2. *The $(N \times 1)$ -vector of log-changes in sectoral prices is*

$$d \ln(\mathbf{p}) = \mathbf{L} \left[-\text{diag}(\boldsymbol{\chi}) \boldsymbol{\varepsilon} + d \ln(\boldsymbol{\phi}) + \text{diag}(\boldsymbol{\iota} - \boldsymbol{\chi}) d \ln(\mathbf{R}) \right], \quad (\text{D.3})$$

where \mathbf{L} denotes the revenue-based Leontief inverse in Definition 2. For given changes in (relative) interest rates, downstream credit shares, $\boldsymbol{\theta}_{\cdot i}$, and the credit-share weighted sectoral sales composition, $dS_{\mathcal{X},i} = \sum_c \theta_{ci} d\mathcal{X}_{ci}$ with $\mathcal{X}_{ci} = \frac{x_{ci}}{q_i}$, changes in sector i 's combined credit wedge as given by Equation (28) are

$$\begin{aligned} d \ln(\phi_i) = & \frac{W\ell_i + \sum_s p_s x_{is}}{R_i} dr_i^b - \frac{AP_i}{R_i} d\rho_i^b(\mathbf{W}_{i:}^P) - \frac{AR_i}{R_i} dr_i^\tau \\ & - \sum_s \frac{(r_i^b - r_s^\tau) AP_{is}}{R_i} d \ln(\theta_{is}) - \sum_c \frac{r_i^\tau AR_{ci}}{R_i} d \ln(\theta_{ci}) - \frac{r_i^\tau}{\phi_i^R} dS_{\mathcal{X},i}, \end{aligned} \quad (\text{D.4})$$

where the change in i 's upstream relative bank rate, $d\rho_i^b$, is as in Proposition 4 with weights

$W_{is}^P = AP_{is}(\sum_s AP_{is})^{-1}$. The sign of the elasticity on upstream credit shares, θ_i , depends on the interest rate differential, $(r_i^b - r_s^\tau)$.

Proof of Lemma D.2. The log-linearization of the optimal price and marginal costs in the Cobb-Douglas economy given by Corollary 1 and Equation (B.21), respectively, yields

$$\hat{p}_i = \sum_s \gamma_{is} \left(\hat{p}_s + \hat{\phi}_{is}^X - \hat{\phi}_i^R \right) + \chi_i \eta_i \left(\hat{W} + \hat{\phi}_i^L - \hat{\phi}_i^R \right) - \chi_i \varepsilon_i + (1 - \chi_i) \left(\hat{R}_i - \hat{\phi}_i^R \right) \quad (\text{D.5})$$

Stacking equations while taking the aggregate wage rate as the numeraire and solving for the vector of sectoral price responses yields Equation (D.3).

The change in the price wedge as defined in Equation (28) follows from substituting the response of credit and revenue wedges in, $\hat{\phi}_i = \sum_s \gamma_{is} \hat{\phi}_{is}^X + \chi_i \eta_i \hat{\phi}_i^L - \hat{\phi}_i^R$, using the results of Lemma D.1 and Equation (D.4) follows. This completes the proof of Lemma D.2. \square

Lemma D.3. The change in the sales shares of sector i is given by

$$d\lambda_i = - \sum_n \left[\tilde{\mathbb{L}}'_{in} - (1 - \tau) \lambda_i \sum_j (1 - \tilde{\chi}_j) \tilde{\mathbb{L}}'_{jn} \right] \lambda_n d \ln(\phi_n^S) - \tau \lambda_i \sum_n \lambda_n d\tilde{\chi}_n \quad (\text{D.6})$$

and changes in the aggregate labor share in GDP are

$$d\Lambda = (1 - \tau) \left[\frac{WL}{PY} \sum_n \left(\sum_j (1 - \tilde{\chi}_j) \tilde{\mathbb{L}}'_{jn} \right) \lambda_n d \ln(\phi_n^S) + \frac{PF}{PY} \sum_n \lambda_n d\tilde{\chi}_n \right], \quad (\text{D.7})$$

where the distortion- and income-adjusted Leontief inverse, $\tilde{\mathbb{L}}$, and sector i 's net cost share, $\tilde{\chi}_i$, are characterized in Definition 2. Log-changes in the sectoral sales wedge are

$$\begin{aligned} d \ln(\phi_i^S) = & \sum_c \frac{p_i x_{ci}}{R_i} \frac{\phi_i^R}{\phi_{ci}^X} dr_c^b - \frac{AR_i}{R_i} dr_i^\tau + \left[\sum_c \frac{AR_{ci}}{R_i} \frac{\phi_i^R}{\phi_{ci}^X} \right] d\rho_i^\tau(\mathbf{W}_{:i}^S) \\ & - \sum_c \left[r_i^\tau + (r_c^b - r_i^\tau) \frac{\phi_i^R}{\phi_{ci}^X} \right] \frac{AR_{ci}}{R_i} d \ln(\theta_{ci}) - \frac{r_i^\tau}{\phi_i^R} dS_{\chi,i}. \end{aligned} \quad (\text{D.8})$$

The sales-weighted sum of changes in sectoral net cost shares is

$$\sum_i \lambda_i d\tilde{\chi}_i = - \frac{L}{L - M} \left\{ \sum_i \lambda_i \left[d \ln(\phi_i^X) - \left(\sum_j \frac{M_j}{R_j} \tilde{\mathbb{L}}'_{ji} \right) d \ln(\phi_i^S) \right] + \frac{M}{PY} d \ln(L) \right\}. \quad (\text{D.9})$$

The log-change in the sectoral cost-share wedge is

$$\begin{aligned} d \ln(\phi_i^X) = & \left[\frac{W \ell_i}{R_i} \frac{1}{\phi_i^L} + \sum_s \frac{p_s x_{is}}{R_i} \frac{(1 + r_s^\tau \theta_{is})}{\phi_{is}^X} \right] dr_i^b - \left[\sum_s \frac{AP_{is}}{R_i} \frac{(1 + r_s^\tau \theta_{is})}{\phi_{is}^X} \right] d\rho_i^b(\mathbf{W}_{:i}^X) \\ & - \sum_s \frac{AP_{is}}{R_i} dr_s^\tau - \sum_s \left[r_s^\tau + (r_i^b - r_s^\tau) \left(\frac{1 + r_s^\tau \theta_{is}}{\phi_{is}^X} + \frac{1}{\phi_i^L} \right) \right] \frac{AP_{is}}{R_i} d \ln(\theta_{is}). \end{aligned} \quad (\text{D.10})$$

The change in the relative downstream trade credit interest rate, $d\rho_i^\tau(\mathbf{W}_{:i}^S)$, and the relative upstream bank interest rate, $d\rho_i^b(\mathbf{W}_{:i}^X)$, are given in Proposition 4 with weights

$$\mathbf{W}_{ci}^S = \frac{AR_{ci}(\phi_{ci}^X)^{-1}}{\sum_c AR_{ci}(\phi_{ci}^X)^{-1}} \quad \text{and} \quad \mathbf{W}_{is}^X = \frac{AP_{is}(1 + r_s^\tau \theta_{is})(\phi_{is}^X)^{-1}}{\sum_s AP_{is}(1 + r_s^\tau \theta_{is})(\phi_{is}^X)^{-1}}, \quad (\text{D.11})$$

respectively. The elasticities wrt. changes in (relative) interest rates, downstream credit shares and in the weighted sectoral sales composition are positive while the sign of the elasticity wrt. upstream trade credit shares depends on the interest rate differential, $(r_i^b - r_s^\tau)$.

Proof of Lemma D.3. (a) Using the results of Section C.1, the log-linearization of the aggregate resource constraint, $R_{N+1} = WL + \sum_i (1 - \tilde{\chi}_i) R_i$, implies that the response of final sales and subsequently nominal GDP are given by

$$L\hat{L} + (1 - \tau) \sum_i \left[(1 - \tilde{\chi}_i) R_i \hat{R}_i - \tilde{\chi}_i R_i \hat{\chi}_i \right] \begin{cases} = R_{N+1} \hat{R}_{N+1} & \text{if } \tau = 0 \\ = PY \hat{P}Y & \text{if } \tau \in [0, 1] \end{cases}. \quad (\text{D.12})$$

Similarly, the log-linearization of the market clearing conditions for sectoral production (9) while substituting for the optimal input demand of i 's customers yields

$$\hat{R}_i = \sum_c \tilde{\gamma}_{ci} \frac{R_c}{R_i} \hat{R}_c + \tilde{\beta}_i \frac{R_{N+1}}{R_i} \hat{R}_{N+1} - \hat{\phi}_i^S, \quad \text{where} \quad \hat{\phi}_i^S = \sum_c \tilde{\gamma}_{ci} \frac{R_c}{R_i} \hat{\phi}_{ci}^X - \hat{\phi}_i^R \quad (\text{D.13})$$

denotes i 's sales wedge response. The typical entries of the credit-wedge adjusted IO-matrix, $\tilde{\Gamma}$, and the vector of final demand shares, $\tilde{\beta}$, are characterized in Definition 2. Stacking equations, substituting the response of final sales using Equation (D.12) and solving for the vector of sales responses yields

$$\hat{\mathbf{R}} = \underbrace{\text{diag}(\mathbf{R})^{-1} \tilde{\mathbb{L}}' \tilde{\beta} L}_{=\boldsymbol{\iota}} \hat{L} - \underbrace{\text{diag}(\mathbf{R})^{-1} \tilde{\mathbb{L}}' \tilde{\beta} (\tilde{\chi} \circ \mathbf{R})'}_{=\Lambda^{-1} \boldsymbol{\iota} (\boldsymbol{\lambda} \circ \tilde{\chi})'} \hat{\tilde{\chi}} - \underbrace{\text{diag}(\mathbf{R})^{-1} \tilde{\mathbb{L}}' \text{diag}(\mathbf{R})}_{=\mathbb{L}'} \hat{\phi}_S \quad (\text{D.14})$$

which can be simplified using $\mathbf{R} = \tilde{\mathbb{L}}' \tilde{\beta} L$. Next, substituting the response of sectoral sales in Equation (D.12) using Equation (D.14) implies that changes in aggregate nominal GDP are

$$\hat{P}Y = \hat{L} - \underbrace{(-\tau + \Lambda^{-1})}_{=(1-\tau)PF(L)^{-1}} (\boldsymbol{\lambda} \circ \tilde{\chi})' \hat{\tilde{\chi}} - (1 - \tau) ((\boldsymbol{\iota} - \tilde{\chi}) \circ \boldsymbol{\lambda})' \mathbb{L}' \hat{\phi}_S \quad (\text{D.15})$$

which follows from collecting terms. Using Equations (D.14, D.15), implies that changes in sector i 's Domar weight, $\boldsymbol{\lambda} = \mathbf{R}(PY)^{-1}$, and in the aggregate labor share, $\Lambda = L(PY)^{-1}$, are then given by Equations (D.6) and (D.7).

(b) In the remainder, I characterize changes in sales wedges and cost shares.

First, substituting the response of wedges in Equation (D.13) using Lemma D.1 and collecting terms yields Equation (D.8). Second, define $\mathbf{m} = [m_1, \dots, m_N]'$ as the vector of management

cost shares in sales, $m_i = M_i(R_i)^{-1}$, and let $M = \sum_i M_i$, where $M_i = W\ell_i^\tau$. The cost share, $\tilde{\chi}_i$, in Equation (25), can alternatively be written as

$$\tilde{\chi}_i = \frac{W\ell_i^\tau}{R_i} + \frac{\eta_i \chi_i}{\phi_i^L} + \sum_s (1 + r_s^\tau \theta_{is}) \frac{\gamma_{is}}{\phi_{is}^X} \quad \text{such that} \quad (\lambda_i \cdot \tilde{\chi}_i) \hat{\chi}_i = -\lambda_i \hat{\phi}_i^X - (m_i \cdot \lambda_i) \hat{R}_i$$

equals the response of the cost share and changes in the sectoral cost-share wedge, $\hat{\phi}_i^X$, are

$$\hat{\phi}_i^X = \frac{W\ell_i}{R_i} \hat{\phi}_i^L + \sum_s \frac{(1 + r_s^\tau \theta_{is}) p_s x_{is}}{R_i} \hat{\phi}_{is}^X - \frac{AP_{is}}{R_i} dr_s^\tau - \frac{r_i^b (1 + r_s^\tau) AP_{is}}{\phi_i^L R_i} \hat{\theta}_{is}.$$

Substituting the responses of credit and revenue wedges in Lemma D.1 and the log-linearized management cost function, $\hat{M}_i = \sum_s \frac{(r_i^b - r_s^\tau)}{1 + r_i^b} \frac{AP_{is}}{M_i} \hat{\theta}_{is}$, yields the response of the sectoral cost-share wedge characterized in Equation (D.10).

Stacking equations, substituting the response of sales (D.14), and collecting terms yields

$$\text{diag}(\lambda \circ \tilde{\chi}) \hat{\chi} = \left[\mathbf{I} - \Lambda^{-1}(\mathbf{m} \circ \lambda) \boldsymbol{\nu}' \right]^{-1} \left\{ -\text{diag}(\lambda) \hat{\phi}_\chi - \text{diag}(\mathbf{m} \circ \lambda) \left[\boldsymbol{\nu} \hat{L} - \mathbb{L}' \hat{\phi}_S \right] \right\}.$$

Since $\boldsymbol{\nu}' \left[\mathbf{I} - \Lambda^{-1}(\mathbf{m} \circ \lambda) \boldsymbol{\nu}' \right]^{-1} = \frac{L}{L-M} \boldsymbol{\nu}'$ the weighted sum of changes in cost-shares is then given by Equation (D.9). This completes the proof of Lemma D.3. \square

Lemma D.4. *For given changes in interest rates, credit shares and labor, the response of sectoral sales is given by*

$$\begin{aligned} \hat{R}_i = & \frac{L}{L-M} \hat{L} + \sum_n \frac{[\mathcal{L}']_{in} R_n}{R_i} \sum_c \frac{r_n^\tau AR_{cn}}{R_n} \hat{\chi}_{cn} \\ & - \underbrace{\sum_n \left\{ [n_{sb}]_{in} \frac{BC_n}{R_n} \right\}}_{=[\mathcal{E}_{sb}]_{in}} dr_n^b + \underbrace{\sum_n \left\{ [n_{s\tau}]_{in} \frac{AR_n}{R_n} \right\}}_{=[\mathcal{E}_{s\tau}]_{in}} dr_n^\tau + \underbrace{\sum_{c,s} \left\{ [n_{s\theta}]_{i,cs} \frac{AP_{cs}}{R_s} \right\}}_{=[\mathcal{E}_{s\theta}]_{i,cs}} \hat{\theta}_{cs} \end{aligned} \quad (\text{D.16})$$

The network effects of the elasticity wrt changes in interest rates and credit shares are

$$[n_{sb}]_{in} = \sum_s \frac{[\mathcal{L}']_{is} R_s}{R_i} \frac{R_n}{R_s} \frac{AP_{ns}^-}{BC_n} \frac{\phi_s^R}{\phi_{ns}^X} - \frac{R_n}{L-M} \left[\frac{W\ell_n}{BC_n} \frac{1}{\phi_n^L} + \sum_s \frac{AP_{ns}^-}{BC_n} \frac{(1 + r_s^\tau \theta_{ns})}{\phi_{ns}^X} \right] \quad (\text{D.17})$$

$$[n_{s\tau}]_{in} = \frac{[\mathcal{L}']_{in} R_n}{R_i} \left[1 - \sum_c \frac{\phi_n^R}{\phi_{cn}^X} \frac{AR_{cn}}{AR_n} \right] - \frac{R_n}{L-M} \left[1 - \sum_c \frac{1 + r_n^\tau \theta_{cn}}{\phi_{cn}^X} \frac{AR_{cn}}{AR_n} \right] \quad (\text{D.18})$$

$$[n_{s\theta}]_{i,cs} = \frac{[\mathcal{L}']_{is} R_s}{R_i} \left[r_s^\tau + (r_c^b - r_s^\tau) \frac{\phi_s^R}{\phi_{cs}^X} \right] - \frac{R_s}{L-M} \left[r_s^\tau + (r_c^b - r_s^\tau) \left(\frac{1 + r_s^\tau \theta_{cs}}{\phi_{cs}^X} + \frac{1}{\phi_c^L} \right) \right] \quad (\text{D.19})$$

with $\mathcal{L}' = \left(\mathbf{I} + \frac{R\mathbf{m}'}{L-M} \right) \tilde{\mathbb{L}}'$, where $\tilde{\mathbb{L}}'$ is defined in Lemma C.1.

Proof of Lemma D.4. Substituting the sum of changes in cost-shares in Equation (D.14) using Equation (D.10) in the proof of Lemma D.3 yields

$$\hat{\mathbf{R}} = \frac{L}{L-M} \iota \hat{\mathbf{L}} - \left(\mathbf{I} + \iota \frac{\mathbf{M}'}{L-M} \right) \text{diag}(\mathbf{R})^{-1} \tilde{\mathbf{L}}' \text{diag}(\mathbf{R}) \hat{\phi}_S + \frac{PY}{L-M} \iota \boldsymbol{\lambda}' \hat{\phi}_\chi$$

where $\mathbf{M} = [M_1, \dots, M_N]$ and $\mathbf{m} = [m_1, \dots, m_N]$ denote the vector of sectoral credit management costs and management cost shares in sales, $m_i = M_i(R_i)^{-1}$, respectively. The elasticity wrt to the weighted sum of sales wedges can be re-written as

$$\left(\mathbf{I} + \iota \frac{\mathbf{M}'}{L-M} \right) \text{diag}(\mathbf{R})^{-1} \tilde{\mathbf{L}}' \text{diag}(\mathbf{R}) = \text{diag}(\mathbf{R})^{-1} \underbrace{\left(\mathbf{I} + \frac{\mathbf{R}\mathbf{m}'}{L-M} \right) \tilde{\mathbf{L}}'}_{=\mathcal{L}'} \text{diag}(\mathbf{R})$$

Using the results of the Proof of Lemma D.3 to substitute, $\boldsymbol{\lambda}' \hat{\phi}_\chi$, and $\text{diag}(\mathbf{R})^{-1} \mathcal{L}' \text{diag}(\mathbf{R}) \hat{\phi}_S$, and collecting terms yields the elasticities of changes in sales wrt to changes in interest rates and credit shares characterized in Equations (D.17) to (D.19). \square

D.2. Proof of Proposition 3

Proof of Proposition 3. Define $\Delta_{is} = (r_i^b - r_s^\tau)$ for ease of notation in the following.

(a) The log-linearization of the BANK INTEREST RATE (7) implies that changes in sector i 's bank interest rate for given financial shocks, changes in labor and sectoral sales shares are

$$\hat{r}_i^b = \sum_{c,s} \mathbb{1}_{s=i} \underbrace{\left[\frac{mZ_i}{r_i^b} \frac{AR_{ci}}{pq_i} \right]}_{=[\mathcal{E}_{b\theta}]_{i,cs}} \hat{\theta}_{cs} + \underbrace{\left[\frac{r_i^z}{r_i^b} \right]}_{=[\mathcal{E}_{bz}]_{ii}} z_i + \underbrace{\left[\frac{mZ_i}{r_i^b} \right]}_{=[\mathcal{E}_{bs}]_{ii}} \hat{S}_{\mathcal{X},i} \quad (\text{D.20})$$

where the elasticities capture the (1) direct effect of financial shocks to the interest premium and (2) indirect effects via changes in the average receivable share. The entries of the diagonal matrix \mathcal{E}_{bs} and the $(N \times N^2)$ matrix, $\mathcal{E}_{b\theta}$, follow from

$$[\mathcal{E}_{bs}]_{ii} = \frac{r_i^z}{r_i^b} \frac{\mu}{\theta_i^z} = \frac{mZ_i}{r_i^b} \quad \text{and} \quad [\mathcal{E}_{b\theta}]_{i,cs} = \mathbb{1}_{s=i} \frac{r_i^z}{r_i^b} \frac{\mu \theta_i^c}{\theta_i^z} \frac{AR_{ci}}{AR_i} = \mathbb{1}_{s=i} \frac{mZ_i}{r_i^b} \frac{AR_{ci}}{pq_i} \quad (\text{D.21})$$

(b) The log-linearized optimal interest rate on TRADE CREDIT (23) follows from $\hat{r}_i^\tau = \hat{m}Z_i + (\hat{B}C_i - \hat{R}_i + \hat{\phi}_i^R)$. To this end, the log-linearization of the change in the bank rate wrt the average trade credit share extended to customers, mZ_i , equals

$$\hat{m}Z_i = \hat{z}_i + (\mu - 1) \hat{\theta}_i^z = \hat{z}_i + \frac{mZ_i'}{mZ_i} \left[\sum_c \frac{AR_{ci}}{pq_i} \hat{\theta}_{ci} + \hat{S}_{\mathcal{X},i} \right], \quad \text{with} \quad \frac{mZ_i'}{mZ_i} = \frac{(\mu - 1)}{\theta_i^z}. \quad (\text{D.22})$$

Next, total bank credit is $BC_i = BC_i^q + M_i$, where $BC_i^q = \sum_s (1 - \theta_{is}) p_s x_{is} + W \ell_i$. Substituting for the optimal input demand and log-linearization yields

$$\begin{aligned} \hat{BC}_i = & \frac{BC_i^q}{BC_i} \hat{R}_i - \left[\frac{W\ell_i}{BC_i} \frac{r_i^b}{\phi_i^L} + \sum_s \frac{AP_{is}^-}{BC_i} \frac{(1-\theta_{is})r_i^b}{\phi_{is}^X} \right] \hat{r}_i^b - \sum_s \left[\frac{AP_{is}^-}{BC_i} \frac{\theta_{is}r_s^\tau}{\phi_{is}^X} \right] \hat{r}_s^\tau \\ & - \sum_s \left[\frac{1+r_s^\tau}{1+r_i^b} - \Delta_{is} \frac{1-\theta_{is}}{\phi_{is}^X} \right] \frac{AP_{is}}{BC_i} \hat{\theta}_{is} \end{aligned} \quad (D.23)$$

which follows from $\hat{M}_i = \sum_s \frac{(r_i^b - r_s^\tau)}{1+r_i^b} \frac{AP_{is}}{M_i} \hat{\theta}_{is}$ and the log-deviations of credit wedges in Lemma D.1. In a last step, I use the results of Lemmata D.1 and D.4 to substitute for the log-change in revenue wedges and sales. Collecting terms implies that changes in sector i 's trade credit interest rate can be decomposed as

$$\hat{r}_i^\tau = -[\mathcal{E}_{\tau b}]_i \hat{r}_i^b + [\mathcal{E}_{\tau\tau}]_i \hat{r}_i^\tau + [\mathcal{E}_{\tau\theta}]_i \hat{\theta} + z_i - [e_{\tau\ell}]_i \hat{L} + [\mathcal{E}_{\tau s}]_i \hat{S}_{\mathcal{X}} \quad (D.24)$$

The elasticities capture the effect of changes in the respective variable on (B1) the marginal effect of changes in the receivable share on i 's bank rate, mZ_i , (B2) direct and (B3) indirect effects via changes in sales on i 's bank loan to production sales ratio, $BC_i(pq_i)^{-1}$, where

$$\begin{aligned} [\mathcal{E}_{\tau b}]_{in} &= \underbrace{\mathbb{1}_{n=i} \left[\frac{W\ell_i}{BC_i} \frac{r_i^b}{\phi_i^L} + \sum_s \frac{AP_{is}^-}{BC_i} \frac{(1-\theta_{is})r_i^b}{\phi_{is}^X} \right]}_{(B2)} - \underbrace{\frac{M_i}{BC_i} [\mathcal{E}_{sb}]_{in}}_{(B3)}, \\ [\mathcal{E}_{\tau\tau}]_{in} &= \underbrace{\mathbb{1}_{n=i} \frac{r_i^\tau AR_i}{R_i} - \frac{AP_{in}^-}{BC_i} \frac{r_n^\tau \theta_{in}}{\phi_{in}^X}}_{(B2)} - \underbrace{\frac{M_i}{BC_i} [\mathcal{E}_{s\tau}]_{in}}_{(B3)}, \quad \text{and} \\ [\mathcal{E}_{\tau\theta}]_{i,cs} &= \underbrace{\mathbb{1}_{s=i} \left[\frac{mZ_i'}{mZ_i} \frac{AP_{ci}}{pq_i} + \frac{r_i^\tau AP_{ci}}{R_i} \right]}_{(B1)+(B2)} - \underbrace{\mathbb{1}_{c=i} \left[\frac{1+r_s^\tau}{1+r_i^b} - \Delta_{is} \frac{1-\theta_{is}}{\phi_{is}^X} \right] \frac{AP_{is}}{BC_i} - \frac{M_i}{BC_i} [\mathcal{E}_{s\theta}]_{i,cs}}_{(B2)+(B3)}, \end{aligned}$$

Elasticities wrt changes in the average sales composition and labor are

$$[\mathcal{E}_{\tau s}]_{in} = \underbrace{\mathbb{1}_{n=i} \frac{mZ_i'}{mZ_i}}_{(B1)} + \underbrace{\left\{ \mathbb{1}_{n=i} - \frac{M_i}{BC_i} \frac{[\mathcal{L}']_{in} R_n}{R_i} \right\} \frac{r_n^\tau}{\phi_n^R}}_{(B2)+(B3)} \quad \text{and} \quad [e_{\tau\ell}]_i = \underbrace{\frac{M_i}{BC_i} \frac{L}{L-M}}_{(B3)}$$

(c) The log-linearization of the optimal TRADE CREDIT SHARES (22) yields

$$\hat{\theta}_{is} = \frac{NP_{is}}{AP_{is}} \left[\frac{r_i^b}{\Delta_{is}} \hat{r}_i^b - \frac{r_s^\tau}{\Delta_{is}} \hat{r}_s^\tau + (\hat{R}_i - \hat{\phi}_{is}^X - \hat{\phi}_i^L) \right], \quad \text{where} \quad \frac{NP_{is}}{AP_{is}} = \frac{\theta_{is} - \theta_{is}^\kappa}{\theta_{is}}$$

denotes sector i 's accounts payable share due to the interest differential in accounts payable owed to supplier s . Substituting the credit-wedge responses and the log-deviation of revenues using Lemma D.1 and D.4, respectively, and collecting terms implies that changes in sector i 's trade credit share obtained from supplier s are

$$\hat{\theta}_e = [\mathcal{E}_{\theta b}]_{is,:} \hat{r}_i^b - [\mathcal{E}_{\theta\tau}]_{is,:} \hat{r}_i^\tau + [\mathcal{E}_{\theta\theta}]_{is,:} \hat{\theta} + [e_{\theta\ell}]_{is} \hat{L} + [\mathcal{E}_{\theta s}]_{is,:} \hat{S}_{\mathcal{X}}. \quad (D.25)$$

The elasticities capture the (C1) direct effect of changes in the respective variable for given expenditures, and their (C2) direct and (C3) indirect effects via changes in i 's sales on i 's intermediate expenditures. Define $TX_{is} = \phi_{is}^X p_s x_{is}$ and $e = is$ for ease of notation. The elasticities wrt changes in interest rates are

$$[\mathcal{E}_{\theta b}]_{en} = \underbrace{\mathbb{1}_{n=i} \frac{r_i^b \tilde{N}P_e}{AP_e}}_{(C1)+(C2)} - \underbrace{\frac{NP_e}{AP_e} [\mathcal{E}_{sb}]_{in}}_{(C3)}, \quad \text{and} \quad [\mathcal{E}_{\theta \tau}]_{en} = \underbrace{\mathbb{1}_{n=s} \frac{r_s^\tau \tilde{N}R_e}{AP_e}}_{(C1)+(C2)} - \underbrace{\frac{NP_e}{AP_e} [\mathcal{E}_{s\tau}]_{in}}_{(C3)},$$

respectively. An entry of the elasticity matrix wrt changes in credit shares are

$$[\mathcal{E}_{\theta \theta}]_{e,nk} = \underbrace{\mathbb{1}_{nk=e} \Delta_e \frac{NP_e}{TX_e}}_{(C2)} + \underbrace{\frac{NP_e}{AP_e} [\mathcal{E}_{s\theta}]_{i,nk}}_{(C3)}, \quad [\mathcal{E}_{\theta s}]_{en} = \underbrace{\frac{NP_e}{AP_e} \frac{[\mathcal{L}']_{in} R_n}{R_i} \frac{r_n^\tau}{\phi_n^R}}_{(C3)}, \quad [e_{\theta \ell}]_e = \underbrace{\frac{NP_e}{AP_e} \frac{L}{L-M}}_{(C3)}$$

denote typical entries of elasticities with respect to changes in average sales composition from financial intermediation and labor. Accounts payable due to interest differentials are given by $NP_e = (\theta_e - \theta_e^\kappa) p x_e$ and $\tilde{N}P_e, \tilde{N}R_e > 0$ are defined as

$$\tilde{N}P_e = \frac{NP_e}{\Delta_e} \left[\frac{1 + r_s^\tau}{1 + r_i^b} - \Delta_e \frac{AP_e^-}{TX_e} \right], \quad \text{and} \quad \tilde{N}R_e = \frac{NP_e}{\Delta_e} \left[1 + \Delta_e \frac{AP_e}{TX_e} \right]. \quad (\text{D.26})$$

The elasticity matrices of changes in revenues with respect to changes in interest rates and credit shares are characterized in Lemma D.4.

(d) Note that $\mathcal{E}_{xy} \equiv \frac{\partial \ln(x)}{\partial \ln(y)}$ for $x, y \in \{\mathbf{r}^b, \mathbf{r}^\tau, \boldsymbol{\theta}\}$. Equation (30) in the main text follows from stacking the individual equations in (a) to (c), and solving for changes in credit costs and shares as a function of financial shocks, the weighted sales composition and labor. \square

D.3. Proofs of Propositions 4 and 5

Proof of Proposition 4. From Proposition 2, changes in allocative efficiency are

$$\frac{\partial \ln(\mathcal{Y})}{\partial \mathcal{X}} d\mathcal{X} = \underbrace{-(1-a)\hat{\Lambda} - \mathbf{a}'\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda}'_\tau \hat{\boldsymbol{\phi}}}_{= d\ln(\Phi_z)} = \Lambda_n \frac{M}{PY} \hat{L} - \left[\boldsymbol{\lambda}'_\tau \hat{\boldsymbol{\phi}} + \boldsymbol{\lambda}'_n \hat{\boldsymbol{\phi}}_s - \Lambda_n \boldsymbol{\lambda}' \hat{\boldsymbol{\phi}}_\chi \right]$$

The second equality follows from substituting changes in the aggregate labor share and the vector of changes in sales shares using Equations (D.7) and (D.6) in Lemma D.3, as well as Equation (D.9), collecting terms and rearranging. The aggregate effects of changes in sales wedges and of the weighted sum of changes in costs shares are given by

$$\boldsymbol{\lambda}'_n = \frac{\partial \ln(\mathcal{Y})}{\partial \ln(\boldsymbol{\phi}_s)} = \left\{ (1-\tau)((\boldsymbol{\iota} - \tilde{\boldsymbol{\chi}}) \circ \boldsymbol{\lambda})' - \mathbf{a}' + \Lambda_n(\mathbf{m} \circ \boldsymbol{\lambda})' \right\} \mathbb{L}' \quad \text{and} \quad (\text{D.27})$$

$$\Lambda_n = \frac{\partial \ln(\mathcal{Y})}{\partial \boldsymbol{\lambda}' d\ln(\boldsymbol{\phi}_\chi)} = \left\{ (1-\tau) \frac{PF}{PY} - a \right\} \frac{PY}{L-M}, \quad \text{respectively.} \quad (\text{D.28})$$

Next, substituting the price-, sales and cost-share wedge responses using Equations (D.4), (D.8) and (D.10), collecting terms and re-indexing, first implies that the change in the upstream relative bank-rate can be written as

$$\begin{aligned} & \lambda_{\tau,i} \frac{AP_i}{R_i} d\rho_i^b(\mathbf{W}_{i:}^P) - \Lambda_n \lambda_i \left[\sum_s \frac{AP_{is}}{R_i} \frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X} \right] d\rho_i^b(\mathbf{W}_{i:}^X) \\ &= \left\{ \lambda_{\tau,i} - \Lambda_n \lambda_i \left[\sum_s \frac{AP_{is}}{AP_i} \frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X} \right] \right\} \frac{AP_i}{R_i} \underbrace{\left\{ dr_i^b - \sum_s \frac{\lambda_{\tau,i} AP_{is} - \Lambda_n \lambda_i AP_{is} \frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X}}{\lambda_{\tau,i} AP_i - \Lambda_n \lambda_i \sum_s AP_{is} \frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X}} dr_s^\tau \right\}}_{= d\rho_i^b(\tilde{\mathbf{W}}_{i:})} \end{aligned}$$

It follows that changes in allocative efficiency due to changes in bank-, trade-credit interest rates and shares can be written as $[\lambda'_\tau \hat{\phi} + \lambda'_n \hat{\phi}_S - \Lambda_n \lambda' \hat{\phi}_\chi] = \dots$

$$= \frac{\partial \ln(\mathcal{Y})}{\partial \mathbf{r}^b} d\mathbf{r}^b - \frac{\partial \ln(\mathcal{Y})}{\partial \boldsymbol{\rho}^b} d\boldsymbol{\rho}^b(\cdot) + \frac{\partial \ln(\mathcal{Y})}{\partial \boldsymbol{\rho}^\tau} d\boldsymbol{\rho}^\tau(\cdot) - \frac{\partial \ln(\mathcal{Y})}{\partial \mathbf{r}^\tau} d\mathbf{r}^\tau - \sum_i \frac{\partial \ln(\mathcal{Y})}{\partial \ln(\theta_{i:})} \hat{\theta}_{i:} - \frac{\partial \ln(\mathcal{Y})}{\partial \mathbf{S}_\chi} d\mathbf{S}_\chi$$

where the output-elasticities wrt changes in sector i 's bank- and trade-credit interest rates

$$\begin{aligned} \frac{\partial \ln(\mathcal{Y})}{\partial r_i^b} &= \lambda_{\tau,i} \frac{W\ell_i + \sum_s p_s x_{is}}{R_i} - \Lambda_n \lambda_i \left[\frac{W\ell_i}{R_i} \frac{1}{\phi_i^L} + \sum_s \frac{p_s x_{is}}{R_i} \frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X} \right] + \sum_s \lambda_{n,s} \frac{p_s x_{is}}{R_s} \frac{\phi_s^R}{\phi_{is}^X} \\ \frac{\partial \ln(\mathcal{Y})}{\partial r_i^\tau} &= \left\{ \lambda_{\tau,i} + \lambda_{n,i} - \Lambda_n \lambda_i \right\} \frac{AR_i}{R_i}, \text{ and the relative up- and downstream rates are} \\ \frac{\partial \ln(\mathcal{Y})}{\partial \rho_i^b} &= \left\{ \lambda_{\tau,i} - \Lambda_n \lambda_i \left[\sum_s \frac{AP_{is}}{AP_i} \frac{(1+r_s^\tau \theta_{is})}{\phi_{is}^X} \right] \right\} \frac{AP_i}{R_i}, \quad \frac{\partial \ln(\mathcal{Y})}{\partial \rho_i^\tau} = \lambda_{n,i} \sum_c \frac{AR_{ci}}{R_i} \frac{\phi_i^R}{\phi_{ci}^X}. \end{aligned}$$

The elasticities wrt changes in credit shares and a sector's sales share composition are

$$\begin{aligned} \frac{\partial \ln(\mathcal{Y})}{\partial \ln(\theta_{is})} &= \left\{ \lambda_{\tau,i} + \lambda_{n,s} \frac{\phi_s^R}{\phi_{is}^X} \frac{R_i}{R_s} - \Lambda_n \lambda_i \left(\frac{1+r_s^\tau \theta_{is}}{\phi_{is}^X} + \frac{1}{\phi_i^L} \right) \right\} \frac{\Delta_{is} AP_{is}}{R_i} \\ &+ \left\{ \lambda_{\tau,s} + \lambda_{n,s} - \Lambda_n \lambda_s \right\} \frac{r_s^\tau AR_{is}}{R_s} \quad \text{and} \quad \frac{\partial \ln(\mathcal{Y})}{\partial S_{\chi,i}} = \left\{ \lambda_{\tau,i} + \lambda_{n,i} \right\} \frac{r_i^\tau}{\phi_i^R} \end{aligned}$$

Equation (32) in the main-text follows. This concludes the proof of Proposition 4. \square

Proof of Proposition 5. Using the first order approximation (FOA) of the credit multiplier in Proposition 3 and Equations (D.20), (D.24) and (D.25) implies that for given labor and the average sales composition of sectors the FOA of the elasticity of *bank* and *trade credit interest rates* wrt to financial shocks are

$$\frac{\partial \ln(r_i^b)}{\partial z_n} \approx [\boldsymbol{\mathcal{E}}_{bz}]_{in} \quad \text{and} \quad \frac{\partial \ln(r_i^\tau)}{\partial z_n} \approx -[\boldsymbol{\mathcal{E}}_{\tau b}]_{i:} [\boldsymbol{\mathcal{E}}_{bz}]_{:n} + [(\mathbf{I} + \boldsymbol{\mathcal{E}}_{\tau\tau})]_{i:} [\boldsymbol{\mathcal{E}}_{\tau z}]_{:n},$$

$$\text{and the elasticity of trade credit shares is } \frac{\partial \ln(\theta_{is})}{\partial z_n} \approx [\boldsymbol{\mathcal{E}}_{\theta b}]_{is,:} [\boldsymbol{\mathcal{E}}_{bz}]_{:n} - [\boldsymbol{\mathcal{E}}_{\theta\tau}]_{is,:} [\boldsymbol{\mathcal{E}}_{\tau z}]_{:n}$$

Using the results of Proposition 3 and D.4 in the online appendix yields

$$\begin{aligned} \frac{\partial \ln(r_i^\tau)}{\partial z_n} &\approx \mathbb{1}_{i=n} - \underbrace{\left[\mathbb{1}_{n=i} \left(\frac{W \ell_i}{BC_i} \frac{1}{\phi_i^L} + \sum_s \frac{AP_{is}^-}{BC_i} \frac{(1 - \theta_{is})}{\phi_{is}^X} \right) \frac{R_i}{BC_i} - \frac{M_i}{BC_i} [n_{sb}]_{in} \right]}_{=[n_{\tau b}]_{in}} \frac{r_n^z BC_n}{R_n} \\ &\quad + \underbrace{\left[\mathbb{1}_{n=i} - \left(\frac{AP_{in}^-}{BC_i} \frac{\theta_{in}}{\phi_{in}^X} \frac{R_n}{AR_n} + \frac{M_i}{BC_i} [n_{s\tau}]_{in} \right) \right]}_{=[n_{\tau\tau}]_{in}} \frac{r_n^\tau AR_n}{R_n} \text{ and} \end{aligned} \quad (\text{D.29})$$

$$\begin{aligned} \frac{\partial \ln(\theta_{is})}{\partial z_n} &\approx \underbrace{\frac{NP_e}{AP_e} \left\{ \mathbb{1}_{n=i} \frac{\tilde{N}P_e}{NP_e} \frac{R_i}{BC_i} - [n_{sb}]_{in} \right\}}_{=[n_{\theta b}]_{en}} \frac{r_n^z BC_n}{R_n} - \underbrace{\frac{NP_e}{AP_e} \left\{ \mathbb{1}_{n=s} \frac{\tilde{N}R_e}{NP_e} \frac{R_s}{AR_s} - [n_{s\tau}]_{in} \right\}}_{=[n_{\theta\tau}]_{en}} \frac{r_n^\tau AR_n}{R_n} \end{aligned} \quad (\text{D.30})$$

Next, I collect and substitute the elasticities corresponding to changes in interest rates in Equation (33) using the expressions derived in the proof of Proposition 4. The elasticity wrt to changes in the bank interest rate of sector i are

$$\begin{aligned} [n_b]_i &\equiv \frac{\partial \ln(\mathcal{Y})}{\partial r_i^b} - \frac{\partial \ln(\mathcal{Y})}{\partial \rho_i^b} - \sum_s \frac{\partial \ln(\mathcal{Y})}{\partial \rho_s^\tau} \frac{AR_{is}/\phi_{is}^X}{\sum_c AR_{cs}/\phi_{cs}^X} = \dots \\ &= \left\{ \lambda_{\tau,i} + \sum_s \lambda_{n,s} \frac{R_i}{R_s} \frac{AP_{is}^-}{BC_i^q} \frac{\phi_s^R}{\phi_{is}^X} - \Lambda_n \lambda_i \left[\frac{W \ell_i}{BC_i^q} \frac{1}{\phi_i^L} + \sum_s \frac{AP_{is}^-}{BC_i^q} \frac{1 + r_s^\tau \theta_{is}}{\phi_{is}^X} \right] \right\} \frac{BC_i^q}{R_i} \end{aligned}$$

The elasticity wrt changes in the trade credit rate is

$$\begin{aligned} [n_\tau]_i &\equiv -\frac{\partial \ln(\mathcal{Y})}{\partial r_i^\tau} + \sum_c \frac{\partial \ln(\mathcal{Y})}{\partial \rho_c^b} \tilde{W}_{ci} + \frac{\partial \ln(\mathcal{Y})}{\partial \rho_i^\tau} = \dots \\ &= \left\{ \sum_c \lambda_{\tau,c} \frac{R_i}{R_c} \frac{AP_{ci}}{AR_i} - \lambda_{\tau,i} + \lambda_{n,i} \left[\sum_c \frac{AR_{ci}}{AR_i} \frac{\phi_i^R}{\phi_{ci}^X} - 1 \right] - \Lambda_n \lambda_i \left[\sum_c \frac{AR_{ci}}{AR_i} \frac{1 + r_i^\tau \theta_{ci}}{\phi_{ci}^X} - 1 \right] \right\} \frac{AR_i}{R_i} \end{aligned}$$

Furthermore, let $[n_\theta]_{is} \equiv \frac{\partial \ln(\mathcal{Y})}{\partial \ln(\theta_{is})}$. Then changes in allocative efficiency following a financial shock to sector n can be written as

$$\frac{\partial \ln(\mathcal{Y})}{\partial z_n} \equiv - \sum_i [n_b]_i \frac{\partial \ln(r_i^b)}{\partial z_n} - \underbrace{\sum_i [n_\tau]_i \frac{\partial \ln(r_i^\tau)}{\partial z_n}}_{\Delta \text{TC-Rates}} + \underbrace{\sum_{i,s} [n_\theta]_{is} \frac{\partial \ln(\theta_{is})}{\partial z_n}}_{\Delta \text{TC-Shares}} \quad (\text{D.31})$$

I now substituting the elasticities of bank- and trade-credit interest rates and credit shares using their FOAs characterized in Equations (D.29) to (D.30). Collecting terms implies that the FOA of changes in output following changes in trade credit rates and shares are given by

$$(\Delta \text{TC-Rates}) = \sum_i [n_\tau]_i \left\{ \mathbb{1}_{i=n} - [n_{\tau b}]_{in} \frac{r_n^z BC_n}{R_n} + [n_{\tau\tau}]_{in} \frac{r_n^\tau AR_n}{R_n} \right\} \quad (\text{D.32})$$

$$(\Delta \text{TC-Shares}) = \sum_{i,s} [n_\theta]_{is} \frac{NP_{is}}{AP_{is}} \left\{ \mathbb{1}_{n=i} \frac{r_n^z \tilde{N}P_{is}}{NP_{is}} - \mathbb{1}_{n=s} \frac{r_s^\tau \tilde{N}R_{is}}{NP_{is}} - [\mathcal{E}_{sr}]_{in} \right\} \quad (\text{D.33})$$

where $[\mathcal{E}_{sr}]_{in} = [\mathcal{E}_{sb}]_{in} \frac{r_n^z}{r_n^b} - [\mathcal{E}_{s\tau}]_{in}$ and the elasticity matrices of sectoral sales following changes in bank, \mathcal{E}_{sb} , and trade credit interest rates, $\mathcal{E}_{s\tau}$, are characterized in Lemma D.4. Evaluating Equations (D.32) and (D.33) around the CRS-case ($\chi = \tau = 1$) such that $M_i = 0$ and $BC_i^q = BC_i \forall i$ yields Equations (36) and (37) in the main text, where $n_b|_{\text{CRS}} = \lambda_\tau$, and $n_\tau, n_\theta|_{\text{CRS}}$ are given by Equation (38) and follow from $\lambda_{n,i}|_{\text{CRS}} = \Lambda_n|_{\text{CRS}} = 0 \forall i$. \square

D.4. Proof of Corollary 4

Corollary 4 in the main text follows immediately from Proposition 5. Similarly, Corollary D.2 derives a sufficient condition for changes trade credit shares to improve allocative efficiency.

Corollary D.2. *Let the aggregate output response to an idiosyncratic financial shock, $z_i > 0$, be given by Proposition 5. If $\max_c \{\mathbf{n}_{\theta,ci}\} > 0$ in equilibrium, then a sufficient condition for changes in trade credit shares to improve allocative efficiency, $\text{FB}_{\theta,i} > 0$, to a first order is*

$$\underbrace{\frac{\min_{s \in \mathcal{S}_i} \left\{ \frac{n_{\theta,is}}{AP_{is}} \right\}}{\max_{c \in \mathcal{C}_i} \left\{ \frac{n_{\theta,ci}}{AR_{ci}} \right\}}}_{\frac{\Delta Y(\Delta i \text{'s TC-Shares})}{\Delta Y(\Delta c \text{'s TC-Shares})}} > \underbrace{\tilde{\eta}_i^\tau + \frac{\max_{c,s} \left\{ \frac{n_{\theta,cs}}{AP_{cs}} \right\}}{\max_{c \in \mathcal{C}_i} \left\{ \frac{n_{\theta,ci}}{AR_{ci}} \right\}} \frac{\sum_c NP_c [\mathcal{E}_{sr}]_{ci}}{r_i^z \tilde{N}P_i}}_{-\frac{\Delta i \text{'s Customers' TC-Shares}}{\Delta i \text{'s TC-Shares}}}, \text{ where } \tilde{\eta}_i^\tau = \frac{r_i^\tau \tilde{N}R_i}{r_i^z \tilde{N}P_i} \quad (\text{D.34})$$

denotes sector i 's interest-adjusted net-lending position in financing cost, with $\tilde{N}P_i = \sum_s \tilde{N}P_{is}$ and $\tilde{N}R_i = \sum_c \tilde{N}R_{ci}$. If $\max_c \{\mathbf{n}_{\theta,ci}\} < 0$ the inequality conditions are reversed.

The condition given by the inequality in (D.34) has a similar interpretation to (40) in the main text. The right-hand side of (D.34) can also be expressed in terms of i 's interest-adjusted net-lending position capturing the relative strength of changes in upstream and downstream credit shares. Intuitively, if sector i extends more trade credit to customers than it depends on external financing, the relative spillover effects on i 's customers will be larger.

E: Quantitative Application

E.1. Calibration Strategy

This section provides a details on the calibration strategy and Table E.1 at the end of this section lists the (average) values of the production and financial parameters used in the benchmark simulations of the model.

E.1.1. Adjustment of IO- and Financial Data

A. Trade Credit.

Shares. Balance sheet data of the unbalanced panel of US Compustat-firms characterized in Appendix A are used to calculate sectoral shares of accounts payable in total input expenditures, θ_{it}^P , and the share of accounts receivable in total revenues, θ_{it}^R .

Dealing with Missing Data and Domestic Non-Market Clearing.

If a sector is missing: (a) all trade credit data, credit shares are set to zero which implies that this sector is neither extending nor receiving trade credit; (b) some observations, I first identify the period with the highest number of consecutive non-missing observations. Using the first and last observation of this period, the median growth rate of trade credit shares in the sample is used to extrapolate the level of the respective share for the remaining observations.

The levels of sectoral accounts payable, AP_{it} , and receivables, AR_{it} , are then calculated using sectoral credit shares, total intermediate expenditures and sales as recorded in the IO-tables. As the model assumes a closed economy, all trade credit relations are between domestic firms. Market clearing for domestic trade credit, $AP_t = \sum_i AP_{it} = \sum_i AR_{it} = AR_t$, is then ensured by subtracting a share, $s_{it}AR_{it}/(\sum_i s_{it}AR_{it})$ of the aggregate excess supply of trade credit ($AR_t - AP_t$) from sectoral accounts receivable, AR_{it} , where $s_{it} = X_{it}/R_{it}$ denotes the sectoral share of exports in sales (excluding sales to FIRE). Sectoral receivable shares in sales are then updated accordingly.

The *Inter-Sectoral Trade Credit Share* from supplier s to customer i is constructed as a (sales) weighted average of the trade credit shares, as suggested in Altinoglu (2021)

$$\theta_{is} = \frac{R_s}{R_i + R_s} \theta_i^P + \frac{R_i}{R_i + R_s} \theta_s^R \quad (\text{E.35})$$

and is non-zero only if both sectors also engage in trade in intermediate inputs.

B. Adjustments of IO-Tables. The model is calibrated using the summary tables on "Use of Commodities by Industries After Redefinitions" provided by the BEA. To ensure an appropriate mapping of the model to the data, adjustments are made as outlined below.

Treatment of Used and Non-Comparable Imports. The dollar value of the row entries on expenditures on "Used Goods" and "Non-Comparable Imports" are redistributed proportionally

across sector i 's suppliers using the expenditure shares on each sector in i 's total intermediate sales. Any negative intermediate expenditures entries are set to zero.

Total Industry and Commodity Output. The difference between total industry and commodity output is added to final demand (consumption, investment and exports) if positive and to imports otherwise such that nominal output produced equals total sales and markets clear.

Final Demand, Imports and Exports. While the model is a closed economy without investment, sectors in the US economy invest and engage in foreign trade. Two observations can be made: (1) The majority of commodities in the US are (a) produced both domestically and imported and (b) used both as intermediate inputs in production and consumed by final demand. (2) Total final uses (consumption, investment and exports) of most sectors exceed imports while in some sectors (e.g. Oil-Sector) total domestic demand can exceed domestic production. In order to take the model to the data, investments and exports are treated as part of domestic demand of the final good producer. In the calibration, I account for foreign trade (imports) in the form a sales residual in order to ensure market-clearing. Note that by simply ignoring the sales residual in the calibration, good markets would not clear in equilibrium such that sectoral demands exceed production. The calibration ensures that the national accounting identity holds as further discussed below.

Inventories. Since the model is static and does not account for the accumulation of inventories, changes in inventories are subtracted from final uses and the dollar value supplied by sector i is then redistributed proportionally across i 's intermediate customers using the sales share of each sector in i 's total intermediate sales as in [Bigio and La'O \(2020\)](#). Following the adjustment of intermediate sales for changes in inventories, total intermediate expenditures and industry output are recalculated for each sector.

Treatment of FIRE. I follow [Bigio and La'O \(2020\)](#) and interpret the production function (1) as the technology at use related to the physical production inputs rather than interest rates, insurance premia or rental rates. As in [Bigio and La'O \(2020\)](#), the expenditures on FIRE-services are treated as part of capital gains and not as intermediate production expenditures. Therefore, the corresponding rows of the IO-tables are reassigned to gross operating profits and the purchases of FIRE-sectors are treated as part of final demand. To avoid double counting, the share of capital gains attributed to FIRE-expenditures is treated as income accruing to foreign households and thus excluded from the calculation of GDP.

C. Labor Costs and Prices. Data on total hours worked and sectoral prices are provided by the Bureau of Labor Statistics ([BLS](#)), where I rely on the MFP- and the LPC-Database. Total hours worked are used to infer an aggregate wage rate from total labor expenditures recorded in the IO-tables. As the chosen numeraire, the wage rate is then used to normalize prices. Sectoral prices are treated as input prices net of any interest cost on trade credit and subsequently used to construct the aggregate price index and real quantities from the IO-tables. The household's

preference parameters are set such that $\epsilon/\psi = 0.4$, and vary around the values $\psi = 0.5$ and $\epsilon = 0.2$ consistent with the literature.

D. Profit Decomposition. The gross operating surplus - GOP - recorded in the IO-tables include capital expenditures (dp), dividend payments ($ni + dv$) and bank interest rate expenditures ($xint$) net of interest-income ($idit$). (see Horowitz and Planting, 2009) It follows total gross profits are $GRP = GOP + idit = dp + ni + dv + xint$ such that $GOP = (1 - \frac{idit}{GRP})GRP$. Using the income statements of the panel of US Compustat-firms, I first calculate the *sectoral shares* of dividend, capital, interest expenditures and interest income in gross profits as

$$s_{it}^{DV} = \frac{ni + dv}{GRP}, s_{it}^K = \frac{dp}{GRP}, s_{it}^{IR} = \frac{xint}{GRP}, s_{it}^{II} = \frac{idit}{GRP}, \text{ and let } s_{it}^{GPR} = \frac{GRP}{R}$$

denote the sectoral share of gross profits in sales.

The *sectoral levels* of dividends, interest payments and capital expenditures are then inferred from the GOP recorded in the IO-table using the previously calculated shares. To this end, sectoral *Interest Income, Taxes and Profits* are first obtained as follows.

- (a) *Negative Gross Operating Surplus.* Since the model does not allow for negative profits, the GOP is set to zero if a negative value was recorded.
- (b) A sector's *Total Interest Income* is derived as $II_{it} = s_{it}^{II} GPR_{it}$ where *Total Gross Profits* follow from $GPR_{it} = GOP_{it} + II_{it} = s_{it}^{GPR} R_{it}$. To address the concern that the magnitude of either variable could be over- or underestimated following the combination of different data sources, both measures are recalculated by winsorizing (85th-percentile) the sectoral ratios of gross profits to gross operating profits, GPR_{it}/GOP_{it} , and of operating costs (labor and total intermediate input expenditures) to gross profits, $(WL_{it} + XE_{it})/GPR_{it}$.
- (c) *Adjustment of Taxes and Dividends.* The imputed interest income is deducted from taxes which are treated as part of dividend payments to households since the model abstracts from taxes. As a result, a sector's total value added is left unchanged.

E. Interest Rates on Bank Credit. To ensure that the bank interest rate is consistent with the imputed interest expenditures for the extreme case that all operating costs need to be financed using bank credit, the following adjustments are made: In a first step, three different measures of the bank interest rate are calculated:

- (1) $r_{0,it}^b$, implied by $IR_{it} = r_{0,it}^b (WL_{it} + XE_{it})$.
- (2) $r_{1,it}^b = \bar{r} + r_{it}^z$, calculated using sectoral GZ-spreads, r_{it}^z , provided by the authors.
- (3) $r_{2,it}^b$, calculated by combining the level of the interest rate inferred from the IO-tables at the beginning of the period, $r_{0,it}^b$, with the growth rate of $r_{1,it}^b$.

In the calibration, I use the second measure of the bank interest rate unless the share of interest expenditures in gross profits exceeds one, in which case measure (3) is used. The imputed bank interest rate then follows from further winsorizing (85th-percentile) both, the share of interest expenditures, $r_{it}^b(WL_{it} + XE_{it})$, in gross profits, GPR_{it} , as well as the growth rate of sectoral bank interest rates in order to reduce the size of outliers. The calibration consistent risk-free rate is then calculated as $r = \min_{i,t} \{r_{it}^b\}$ such that the implied risk-premia are $r_{it}^z = r_{it}^b - r$.

F. Accounting for Equilibrium Capital Expenditures and Imports.

In the calibration, *capital* is re-introduced into the production function (1) in the main text to allow for a more accurate mapping of the model to the data. The model remains static while the production function is modified to include capital set equal to its equilibrium level that follows from the firm's optimization problem when firms own and invest in their capital stock by purchasing the final good for investment, $i = k' - (1 - \delta)k$. Omitting sectoral indices for simplicity, the firm solves

$$V(s, k) = \max_{\nu, k'} \pi - P(k' - (1 - \delta)k) + \mathbb{E}_t[m'V(s', k')] \quad \text{s.t. } q = \left(A(k^\alpha \ell^{1-\alpha})^\eta X^{1-\eta}\right)^\chi,$$

Equations (5), (6), (7), the feasibility constraints on trade-credit shares and non-negativity constraints for all variables. Since the stochastic discount factor is $m' = \beta^{t+1} C_{t+1}^{-\epsilon}$, the capital euler equation is then given by

$$P = \beta \mathbb{E} \left[\left(\frac{C}{C'} \right)^\epsilon \left(\alpha \eta \chi \frac{R'}{k'} + P'(1 - \delta) \right) \right] \quad \text{such that} \quad \frac{1 - \beta(1 - \delta)}{\beta} Pk = \alpha \eta \chi R \quad (\text{E.36})$$

in equilibrium with $i = \delta k$, and $(1 - \beta(1 - \delta))\beta^{-1} = r^k$ equals the real return on capital. Due to the static nature of the model, it is assumed that capital never depreciates, $\delta = 0$. Equilibrium capital income/expenditures are thus a share of revenues, $\alpha \eta \chi$. The Cobb-Douglas production technology thus implies that the sectoral expenditure shares on capital, labor, intermediate inputs, management costs and dividends, $dv_i = \pi_i - Pr_i^k k_i$, are as follows

$$\underbrace{\sum_s \phi_{is}^X p_s x_{is}}_{(1-\eta_i)\chi_i} + \underbrace{\phi_i^L W \ell_i}_{(1-\alpha_i)\eta_i \chi_i} + \underbrace{\phi_i^L W \ell_i^\tau}_{(1-\chi_i)} + \underbrace{Pr_i^k k_i}_{\alpha_i \eta_i \chi_i} = \sum_c (1 + r_i^\tau \theta_{ci}) p_i x_{ci} + p_i c_i - M_i^* = R_i.$$

The second equality shows that sectoral sales equal the sum of revenues from final and intermediate sales, interest income from extending trade credit to customers net of sectoral imports, M_i^* . The latter effectively constitutes a sales residual so ensure that sectoral expenditures equal sales and markets clear. Revenues of the final good producer stem from selling the final good to households and the financial sector for consumption and to firms as an investment good, $R_{N+1} = PF = P(C + H) + PI$. Note that the latter revenues are zero due to the assumption that capital never depreciates with $\delta = 0$.

Consolidating the household's, financial sector's and firms' budget constraints implies that total

income in the economy is

$$\sum_i \left\{ WL_i + \pi_i + r_i^b \underbrace{\left[WL_i + \sum_s (1 - \theta_{is}) p_s x_{is} \right]}_{BC_i} \right\} = R_{N+1} - M^*$$

where $L_i = \ell_i + \ell_i^T$, and the last equality follows from the national accounting identity with $M^* = \sum_i M_i^*$. It follows that by taking into account the sales residual, the $(N \times 1)$ -vector of sectoral sales can be written as

$$\mathbf{R} = \tilde{\mathbf{L}}' \left[\tilde{\beta} \left(1 + \frac{M^*}{WL} \right) WL - M^* \right] \quad \text{where} \quad \tilde{\mathbf{L}}' = [\mathbf{I} - \tilde{\Gamma}' - \tilde{\beta}(\boldsymbol{\iota} - \tilde{\chi})']^{-1}$$

as characterized in Definition 2 in the main text such that $(1 - \tilde{\chi}_i)R_i = \pi_i + r_i^b BC_i$.

Total Value Added and Final Demand Adjustments involve that the following components need to be excluded from the calculation of value added as they are not part of total domestic income: (1) the sales residual for selected sectors treated as imports, M_i^* , (2) expenditures on and intermediate sales to FIRE-services which have been re-assigned to capital income and final demand, respectively, and (3) the income of the financial sector from interest payments on bank loans, $\sum_i r_i^b BC_i$. It follows that total value added in the economy is thus given by

$$PY = PC = PF - \left[M^* + \underbrace{\sum_i r_i^b BC_i + \text{FIRE}_i}_{=PH} \right] = WL + \sum_i \pi_i - \tilde{\kappa}_i Pr_i^k k_i$$

where FIRE_i is assumed to be a constant fraction, $\tilde{\kappa}_i$, of capital income $Pr_i^k k_i = \alpha_i \eta_i \chi_i R_i$.

E.1.2. Financial Parameters

The Parameter of Risk Premium (7) governing its convexity in the receivable share, μ , is calibrated by estimating the Equation (E.38) at the aggregate level with OLS. To obtain an estimate that is more reflective of the banking technology of the entire economy before the Great Recession, I rely on aggregate [US Flow of Funds Data](#) on accounts receivable including consumer credit (BOGZ1FL103070000Q), depository institution loans (BLNECLBSNNCB) and sales (BOGZ1FA106030005) of all non-financial corporate businesses at a quarterly frequency¹ over the sample period 1997Q1-2007Q4 rather than the sample of Compustat firms characterized in Appendix A.

$$\ln(r_t^z) = \beta_0 + \beta_1 D_{crs} + \mu \ln(d_t + \theta_t^c) + \epsilon_t \quad (\text{E.37})$$

The variable θ_t^c denotes the aggregate share of total accounts receivables in sales and d_t equals the ratio of bank loans to total sales using the corresponding balance sheet items. The data-

¹While I rely on quarterly data to increase the sample size for estimation, a similar estimate was obtained using the same data at an annual frequency.

counterpart for the risk premium, r_t^z , is the aggregate credit spread calculated in [Gilchrist and Zakrajšek \(2012\)](#). Lastly, D_{crs} is a dummy variable equal to one if the US-economy is in a recession according to the NBER business cycle dates (2001Q1-Q4, 2007Q4 - 2009Q2) and zero otherwise. The estimated coefficients and corresponding standard errors in parentheses are reported in the following

$$\ln(r_t^z) = \underbrace{1.771^{***}}_{(0.404)} + \underbrace{0.291^{**}}_{(0.146)} D_{crs} + \underbrace{1.356^{***}}_{(0.501)} \ln(d_t + \theta_t^c) + \epsilon_t \quad (\text{E.38})$$

While the crisis dummy is significant at a five percent level (**), the coefficient of interest, μ , has a p-value of 0.01 and is significant at a one percent level (***).

In the quantitative exercises, I assume that banks in either economy with and without trade credit have access to the same banking technology, such that their monitoring cost function defined in [Appendix B](#) exhibits the same degree of convexity in the aggregate bank loan to sales ratio, d . Since I rely on financial data of Compustat firms to impute equilibrium sectoral trade credit shares, the parameter d is calibrated as follows. In particular, I impose that the ratio of aggregate accounts receivable to depository institutions loans of the sample Compustat firms (CS) in [Appendix A](#) with no missing observations is equal to that of all non-financial US corporations from [US Flow of Funds Data \(FF\)](#). The parameter $d = \frac{AR(\text{CS})}{R(\text{CS})} \left(\frac{AR(\text{FF})}{BC(\text{FF})} \right)^{-1}$ is then set to its pre-crisis (2004-07) average, where $d_{04-07} = 0.03$.

The Parameters of the Credit Management Cost Function (6) are calibrated in three steps: (1) First, a sector's share of total accounts payable in total intermediate cost of production excluding interest rate payments, θ_i , is calculated using [Equation \(22\)](#) and imposing $\kappa_{0,is} = \kappa_0$, $\kappa_{1,is} = \kappa_1$ and $\bar{\theta}_i^s = \bar{\theta}_0^s \forall i, s$, such that

$$\underbrace{\frac{\sum_s \theta_{is} p_s x_{is}}{\sum_s p_s x_{is}}}_{\theta_i} = \underbrace{\left[\bar{\theta}_0^s - (\bar{\theta}_0^s)^2 \frac{\kappa_0}{\kappa_1} \right]}_{\beta_0} + \underbrace{\left[\frac{(\bar{\theta}_0^s)^2}{\kappa_1} \right]}_{\beta_1} \underbrace{\left[\frac{\sum_s (p_s x_{is})^2 (r_i^b - r_s^\tau)}{(\sum_s p_s x_{is})^2} \right]}_{(b) \cdot (a) = pq_i^e} \underbrace{\left[\frac{\sum_s p_s x_{is}}{(1 + r_i^b)} \right]}_{(E.39)}$$

The variable pq_i^e denotes the effective net-interest expenditures and equals the product of (a) the discounted intermediate cost of production excluding interest rate payments and (b) the sector-specific Credit-Hirschman-Herfindahl Index (HHI). Similar to the HHI-index measuring the degree of monopoly power in an industry ([Shepherd, 1987](#)), the Credit-HHI captures a sector's concentration of net-interest rate costs of production. While the sign of the index depends on the relative cost of bank and trade credit, a higher absolute value implies a higher dependency on a particular supplier. Since actual data on the cost of credit are not available, the data-counterparts of θ_i and pq_i^e are obtained by mapping the model to the data. Note that, while θ_i is stationary, pq_i^e is a non-stationary variable such that the detrended² effective net-interest expenditures are used in the estimation of [Equation \(E.39\)](#).

²The variable, pq_i^e , is first detrended using an hp-filter with a smoothing constant of 6.25 ([Ravn and Uhlig, 2002](#)) and then normalized by adding the cyclical component to the sectoral time-mean of pq_i^e .

(2) In a second step, Equation (E.39) is estimated by OLS using a panel of 45 sectors from 1997-2007, while controlling for time and sector fixed effects.

(3) The link-specific cost parameters, $\kappa_{0,is}$ and $\kappa_{1,is}$ are then retrieved by first multiplying Equation (E.39) by (θ_{is}/θ_i) and matching the adjusted estimated coefficients of the net-interest expenditures on intermediate production expenditures in Equation (E.39) and Equation (22)

$$\frac{(\bar{\theta}_i^s)^2}{\kappa_{1,is}} \approx \hat{\beta}_1 \cdot \frac{\theta_{is}}{\theta_i} \quad \text{such that} \quad \kappa_{1,is} = (\bar{\theta}_i^s)^2 \cdot \frac{\theta_i}{\theta_{is}\hat{\beta}_1} \quad (\text{E.40})$$

The link-specific cost parameters, $\kappa_{1,is}$ are further winsorized (85th-percentile) to reduce the effect of outliers. The parameters, $\kappa_{0,is}$, and $m_{0,i}$, are calculated as residuals using Equations (22) and (6). I ensure that the quadratic adjustment cost parameters, $\kappa_{1,is}$, and the fixed cost component, $m_{0,i}$, are strictly positive, whereas the linear cost parameters, $\kappa_{0,is}$, may take on both positive and negative values.

Table E.1: Calibrated Parameters

	VAR	Description	All	NL	NB	p-val		VAR	Description	All	NL	NB	p-val
Production	α	Capital Share	0.337	0.397	0.280	0.006	Financial	μ	BC - Convex	1.35			
	η	Value Added	0.460	0.397	0.521	0.015		d	BC - Agg. MCost	0.03			
	χ	DRS	0.827	0.820	0.834	0.645		r	BC - RFRate	0.004			
	ω	Intermediate	0.025	0.025	0.025	0.839		m_0	CMCost - Fixed	0.293	0.126	0.453	0.080
	β	Final	0.022	0.009	0.035	0.026		κ_0	CMCost - Linear	0.157	0.124	0.189	0.093
	ϵ	Productivity	3.116	2.826	3.393	0.228		κ_1	CMCost - Quadratic	0.167	0.153	0.180	0.000
	k	Capital	13.31	10.82	15.68	0.307		$\bar{\theta}^s$	Av.TC-Demand	0.101	0.110	0.093	0.013
HH	ϵ	Income Elasticity	0.22						# Observations	45	22	23	
	ψ	Inv. Frisch Elasticity	0.55										

Note: This table reports the (2004-07) average of the aggregate and the mean of the cross-sectional production and financial parameters, capital (in thousands of units) and productivity levels used in the model simulations. Using Definition 1, I divide sectors into net-lenders (NL) and net-borrowers (NB) based on the median of their 2004-07 average net-lending position. The columns (NB) and (NL) then report the mean of the respective parameter for each subgroup of sectors and the last column reports the p-values for the differences in means test

E.2. Mapping of Model Equilibrium to Data

The iterative procedure outlined in Algorithm 1 is a rough sketch of the steps involved in calculating the equilibrium of the model economy.

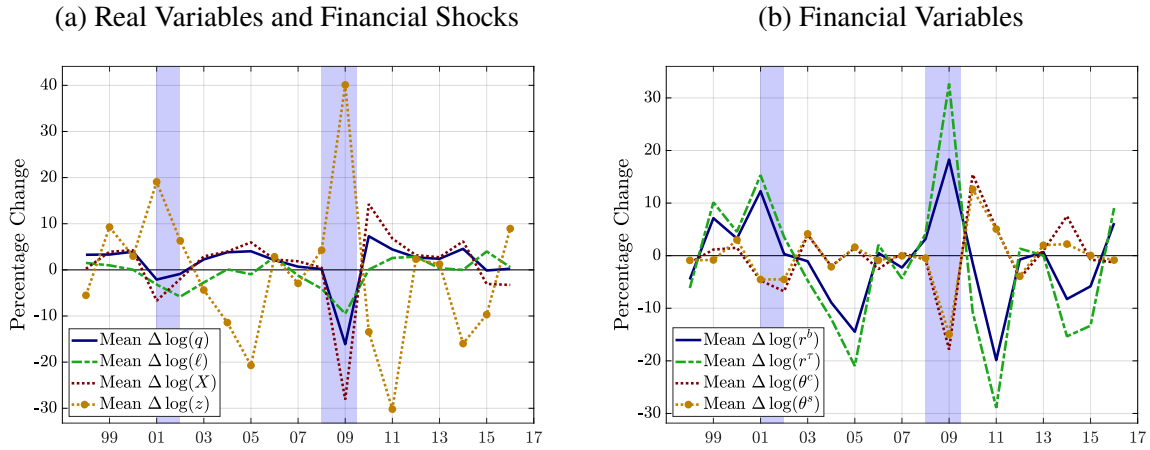
Algorithm 1 Calibration Steps

- 1: Adjustment of Nominal IO-Tables and Credit Network (see Section E.1.1)
 - 2: Calculation of Risk Premium, r^z (see Section E.1.1, E)
 - 3: Initial Guess of Bank Interest Payments, and
 - 4: Production Parameters, $\mathcal{P}_0 = \{\chi, \eta, \alpha, \Gamma, \beta\}_i$
 - 5: **while** $|\mathcal{P}_i - \mathcal{P}_{i-1}| > \epsilon$ **do**
 - 6: Initial Guess of Quantity Shares (\mathcal{X}_0 with $\mathcal{X}_{cs,0} = x_{cs}/q_s$)
 - 7: **while** $|\mathcal{X}_{cs,i} - \mathcal{X}_{cs,i-1}| > \epsilon$ **do**
 - 8: Calculate Equilibrium Financial Variables
 - 9: Calculate Equilibrium Nominal Value Added
 - 10: Calculate Equilibrium Prices and Quantities
 - 11: Calculate Productivity and Financial Shocks (Residuals)
 - 12: Update Quantity Shares, \mathcal{X}_i
 - 13: **end while**
 - 14: Update Production Parameters, \mathcal{P}_i
 - 15: **end while**
 - 16: Calibration of Parameters of Credit Management Cost Function (see Section E.1.2)
-

E.3. Business Cycle Statistics

Following the period-by-period mapping of the equilibrium of the model to the US economy in Section 4 in the main text, Figure E.1 and Table E.2 document business cycle statistics for selected real and financial variables. Panel (a) plots the sectoral mean of the implied financial shock and of the log-change of selected production inputs across time. Panel (b) depicts the average log-changes in the interest rates on bank and trade credit as well as in the trade credit shares. During the 2008-09 recession, the implied shock to interest premia rose by 40.1% and lead to an increase of bank and trade credit interest rates by 18.3% and 32.7%, respectively. Average sectoral output declined by approximately 16.1% caused by a drop in labor and the composite intermediate input by 9.5% and 28.1%, respectively. At the same time, the average trade credit share extended to customers declined by 17.8% and the average share of intermediate expenditures obtained on trade credit dropped by 15.5%.

Figure E.1: Data - Mean Changes



Note: This figure plots the log-change in percent of the time series of selected real and financial variables calculated based on the data discussed in Section 4 from 1997-2016. Panel (a) plots the sectoral mean of the implied financial shock (z_i), the log-change of real output (q_i), labor (ℓ_i) and the intermediate input composite (X_i). Panel (b) plots the average log-change in the interest rate on bank (r_i^b) and trade credit (r_i^r) as well as in the trade-credit shares ($\theta_i^c, \theta_i^s = \sum_s \theta_{is}/N$).

Table E.2 reports the cross-sectional mean of the standard deviation of log-changes in the sectoral variables of interest as well as the within sector correlation between (a) log-changes in output and (b) log-changes in the cost of bank credit and the remaining real and financial variables. In addition, the sample is split into net-lenders (NL) and net-borrowers (NB) based on the median of their 2004-07 net-lending position in Definition 1 and calculated from mapping the model equilibrium to data. A sector is counted as a net-lender if its net-lending position is above the median net-lending share.

The business-cycle statistics of *sectoral output*, *labor* and *the intermediate composite* are similar to those reported in Bigio and La'O (2020): Over the sample period, 1997-2016, labor demand is on average similar or slightly less volatile whereas the demand for the composite intermediate input is more volatile than output. Unsurprisingly, log-changes in output are pos-

itively correlated with changes in production inputs. The difference in means test suggests that there is no significant difference in the volatility or output-correlation between net-borrowing and net-lending sectors. The average *trade-credit* share extended to customers, θ^c , and obtained from suppliers, θ^s , are negatively correlated with the cost of bank finance: While the interest rates on bank and trade credit comove strongly, the latter exhibits a higher standard deviation. This relates to the relative volatility of accounts payable and liabilities and the observation that firms shifted their borrowing portfolio towards bank finance in response to an increase in credit market frictions documented in Section 2. The correlation between the cost of bank and trade credit seems to be significantly higher for the group of sectors classified as net-borrowers.

Table E.2: Data - Time-Series Correlation

(a) Real Variables						(b) Financial Variables					
STDV	VAR	All	NL	NB	p-val	STDV	VAR	All	NL	NB	p-val
	q	0.069	0.071	0.067	0.743		r^b	0.117	0.113	0.121	0.611
	ℓ	0.065	0.078	0.052	0.281		r^τ	0.197	0.199	0.195	0.899
	X	0.141	0.151	0.131	0.541		θ^c	0.119	0.121	0.117	0.770
	z	0.190	0.200	0.180	0.319		θ^s	0.080	0.094	0.067	0.047
CORR	(q, ℓ)	0.557	0.477	0.634	0.169	CORR	(r^b, r^τ)	0.893	0.845	0.940	0.012
	(q, X)	0.807	0.828	0.787	0.487		(r^b, θ^c)	-0.227	-0.231	-0.224	0.940
	(q, z)	-0.491	-0.544	-0.441	0.124		(r^b, θ^s)	-0.313	-0.279	-0.345	0.395
	#OBS	45	22	23			#OBS	45	22	23	

Note: This table reports the time-mean of the standard deviation and the correlation with output and the bank interest rate of the log-change of output (q_i), labor (ℓ_i), the intermediate input composite (X_i), the interest rate on bank (r_i^b) and trade credit (r_i^τ), the trade-credit shares ($\theta_i^c, \theta_i^s = \sum_s \theta_{is}/N$). The first column reports the business cycle statistics for the entire sample. The second and third column report the same statistics for a subgroup of sectors based on the net-lending position in Definition 1. The p-values for the differences in means between the two groups are reported in the last column.

E.4. Additional Simulations

Management Costs. Columns (1-3a'') in Table E.3 present the output responses of similar exercises (1-3a) in the main text, comparing the results in the benchmark economy to those in an equivalent, bank finance-only economy without any credit management costs ($m_i = 0 \forall i$). While the results in Column (1a'') follow from the aggregate imputed financial shock, Columns (2,3a'') consider a symmetric shock to the top five (2a'') net-lenders and (3a'') net-borrowers.

Symmetry. Column (4) in Table E.3 compares the log-change in aggregate output to the same aggregate financial shock in the benchmark case and in an equivalent economy with symmetric credit shares in equilibrium. Column (5) presents the output response in the benchmark economy to the imputed financial as well as to a symmetric shock calculated as the average of the imputed idiosyncratic shocks.

Corollary D.2. Lastly, I also quantify the sufficient condition (D.34) that illustrates when changes in trade credit shares improve allocative efficiency in response to idiosyncratic financial shocks. Using the 2004-07 average of the data-counterparts of the respective elasticities, I evaluate the inequality condition (D.34) for each sector. Column Cor.D.2 then reports the share of net-lenders (NL) and net-borrowers (NB) that satisfy the condition. As evident from Table E.3, the sufficient condition for changes trade credit shares to improve allocative efficiency is met by about half of the sectors in either sample.

Table E.3: Counterfactual Simulations

$\Delta\%Y_{09}$	(1a'')	(2a'')	(3a'')	(4)	(5)	$\frac{\sum_i \mathbb{1}_i}{N} \%$	Cor. D.2
E(θ)	-1.010	-0.050	-0.129	-1.010	-1.010	NB	56.5
CF	-0.524	-0.022	-0.124	-1.101	-1.095	NL	45.5
\mathcal{M}	1.927	2.250	1.043	0.918	0.923	p-val	0.469

Note: This table reports the simulated log-change in percent of aggregate output (Y) to shocks to sectoral interest premia in the benchmark, $E(\theta)$, and selected counterfactual economies as well as the resulting trade credit-multiplier defined in Definition 4. The last column reports the percentage share of sectors in each group that satisfy the smoothing condition ($\mathbb{1} = 1$) for changes in trade credit shares in Corollary D.2 as well as the p-value of a difference in means test.

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